Precise relaxation methods in image processing

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Variational motion estimation

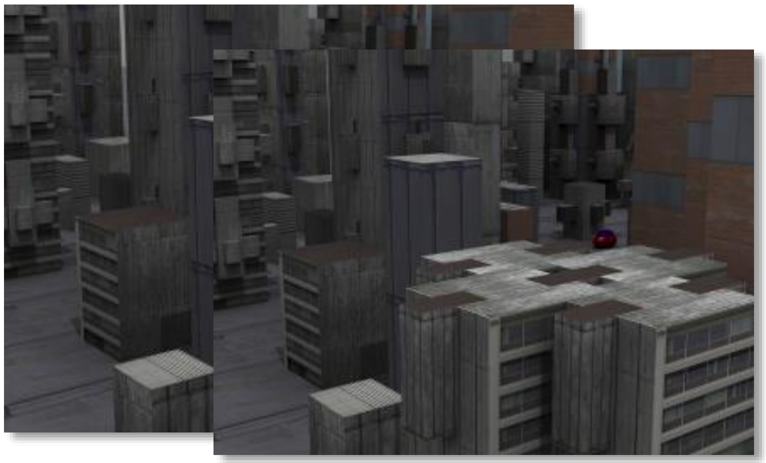


image sequence

optical flow/registration

$$\min_{u:\Omega\to\mathbb{R}^2} \int_\Omega D(I^1_\sigma(x), I^2_\sigma(x+u))dx + \lambda \int_\Omega d\|Du\|$$

u vector field (optical flow, image registration) or scalar (disparity, stereo)

Images: Lellmann, Strekalovskiy, Kötter, Cremers '13

Variational methods

We reconstruct the unknown data *u* from the measurements *b* by minimizing the energy

$$\min_u \{D(T(u); b) + R(u)\}$$

Intuitive (what do we want) and modular (reusable)

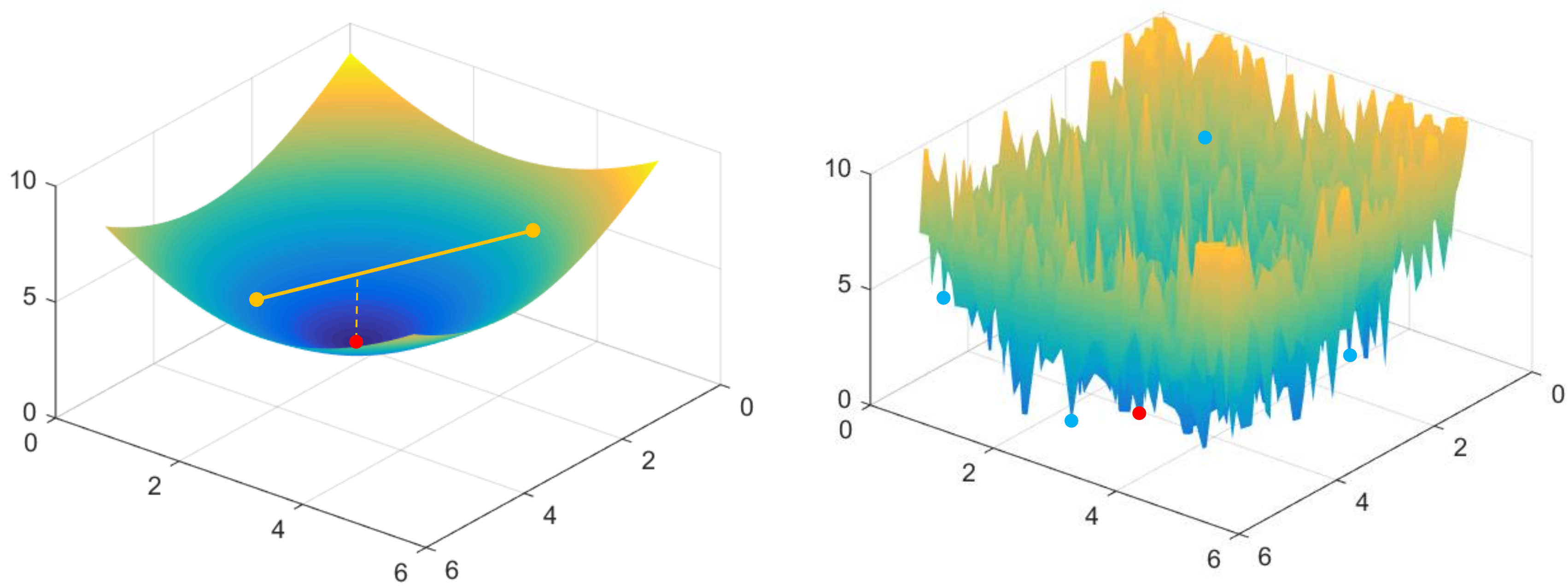
In practice: often

$$\min_{u:\Omega\to X} \int_\Omega \rho(x, u(x))dx + \lambda \int_\Omega \sigma(\nabla u)dx$$

Tikhonov '63

Convexity

Convexity assures that every local minimizer of the energy is also a global minimizer



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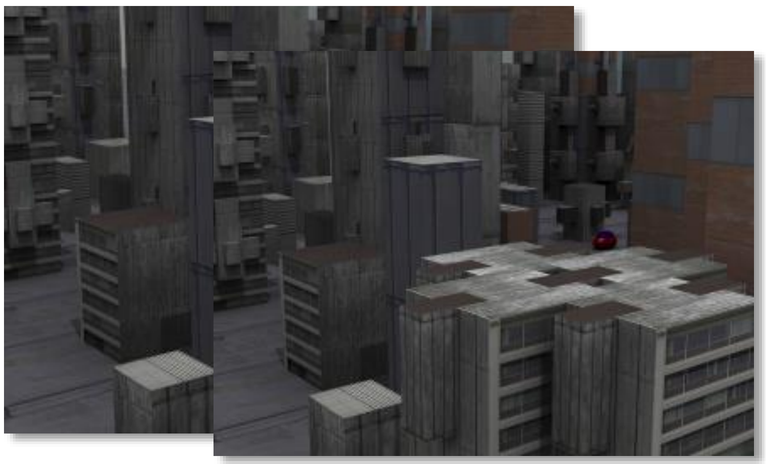

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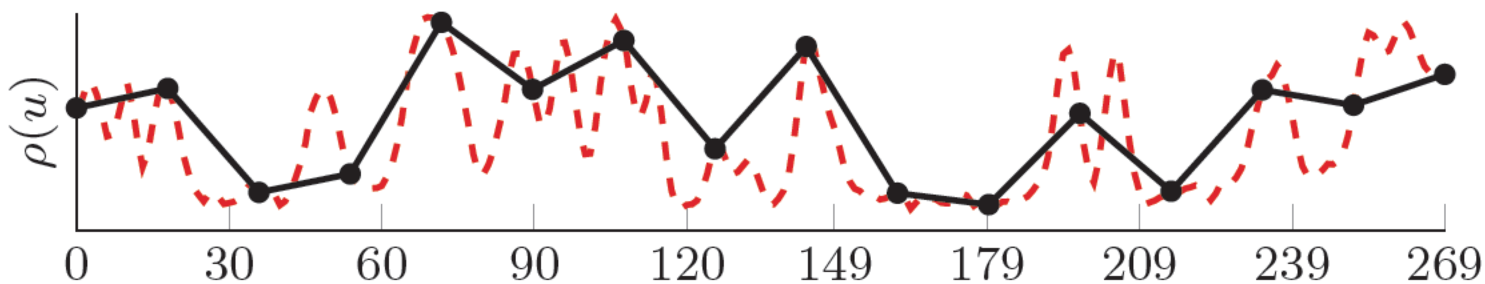
Non-convexity

- Real-world disparity estimation/depth from stereo:



- Data-dependent nonconvexity at every point

Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Exact convex relaxation

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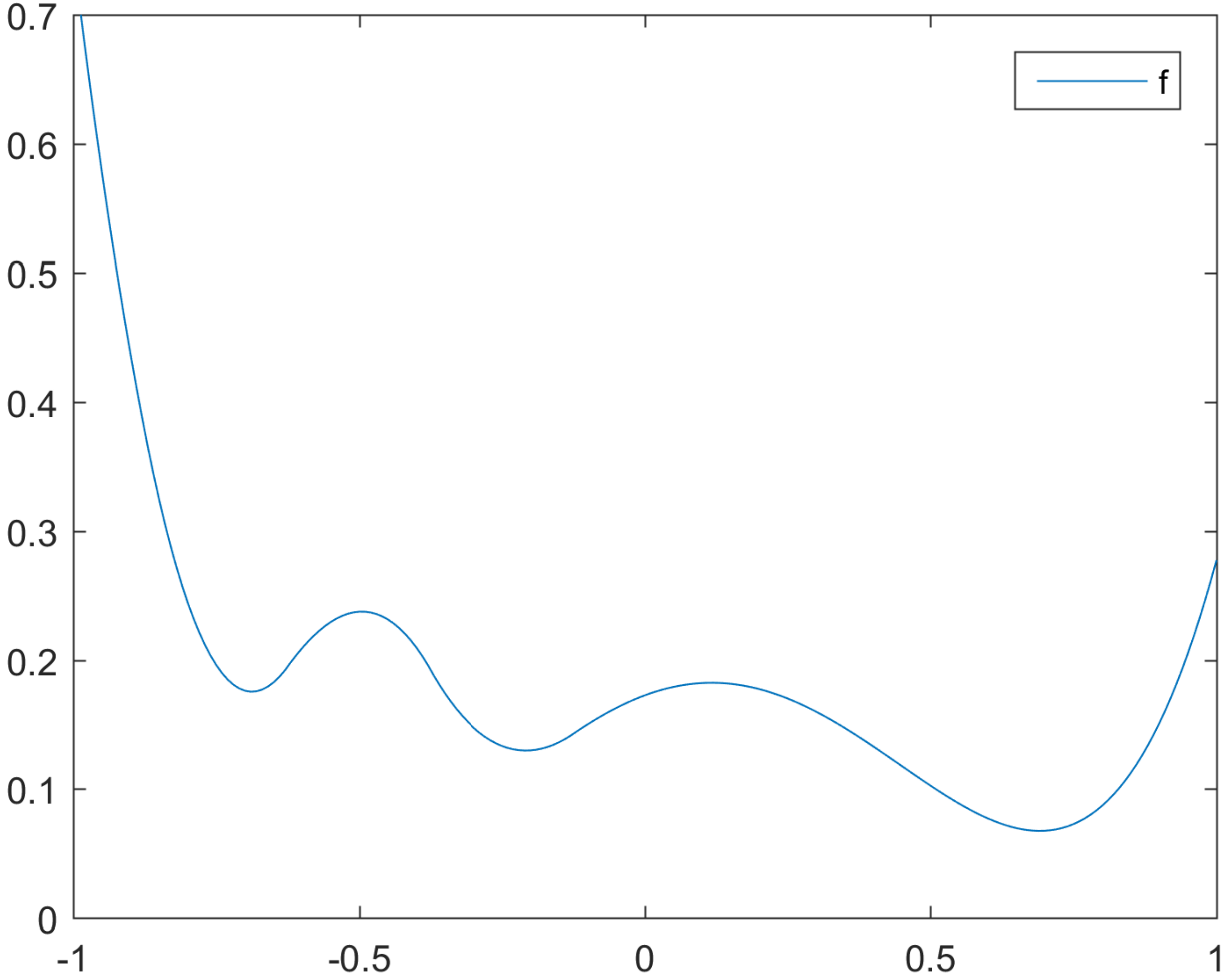

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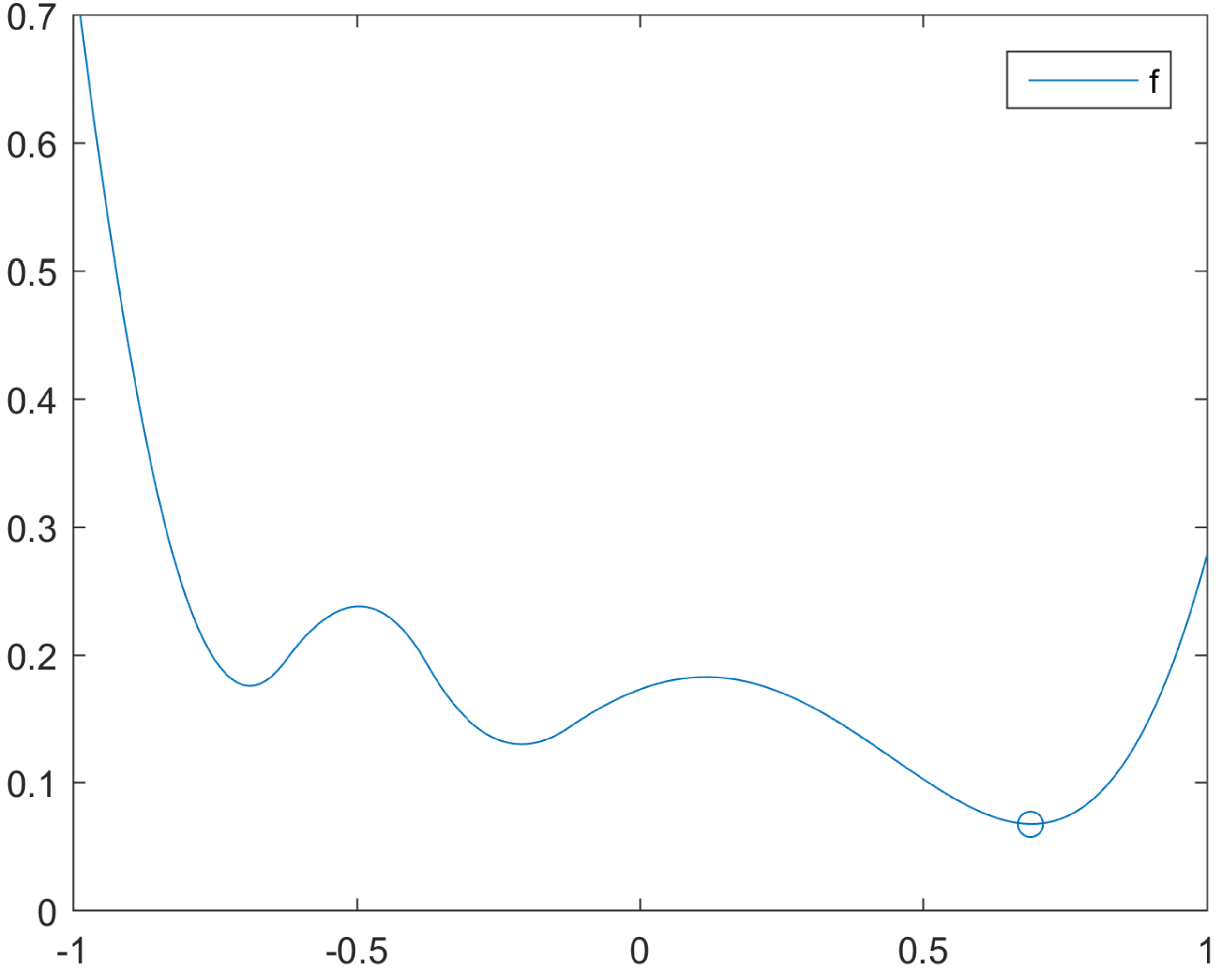

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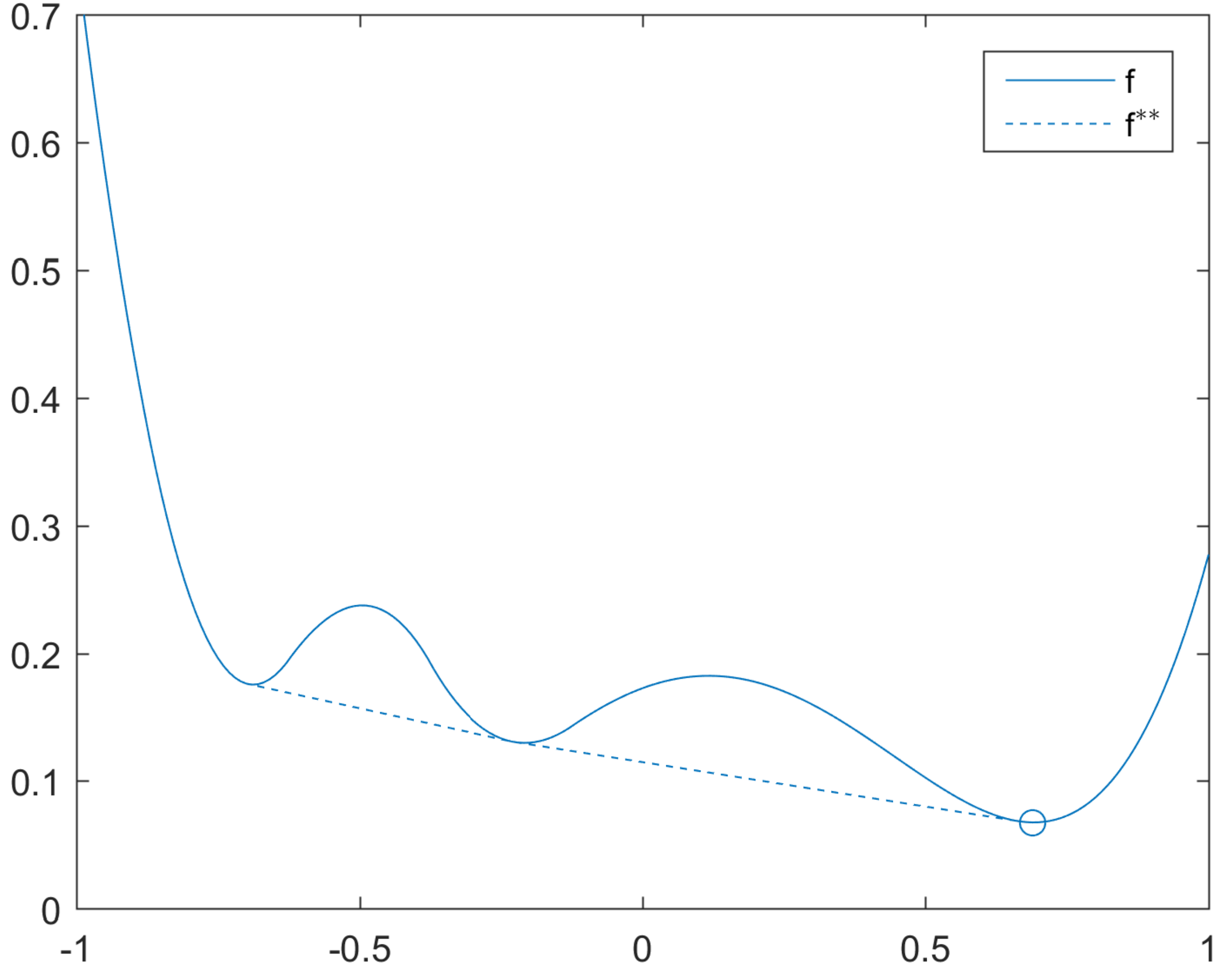

How to find the convex envelope

For $f : \mathbb{R}^n \to \mathbb{R} \cup \{\pm\infty\}$, the *Legendre-Fenchel conjugate* f^* is

$$f^*(v) := \sup_{u \in \mathbb{R}^n} \langle u, v \rangle - f(u).$$

For any function f its *convex envelope* is the *biconjugate* $(f^*)^*$.

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$$\min_{u:\Omega \to \mathbb{R}} f(u) \quad \rightsquigarrow \quad \min_{u:\Omega \to \mathbb{R}} f^{**}(u)$$

Drawback

This is generally as hard as
minimizing the original energy!

Objectives are often sums

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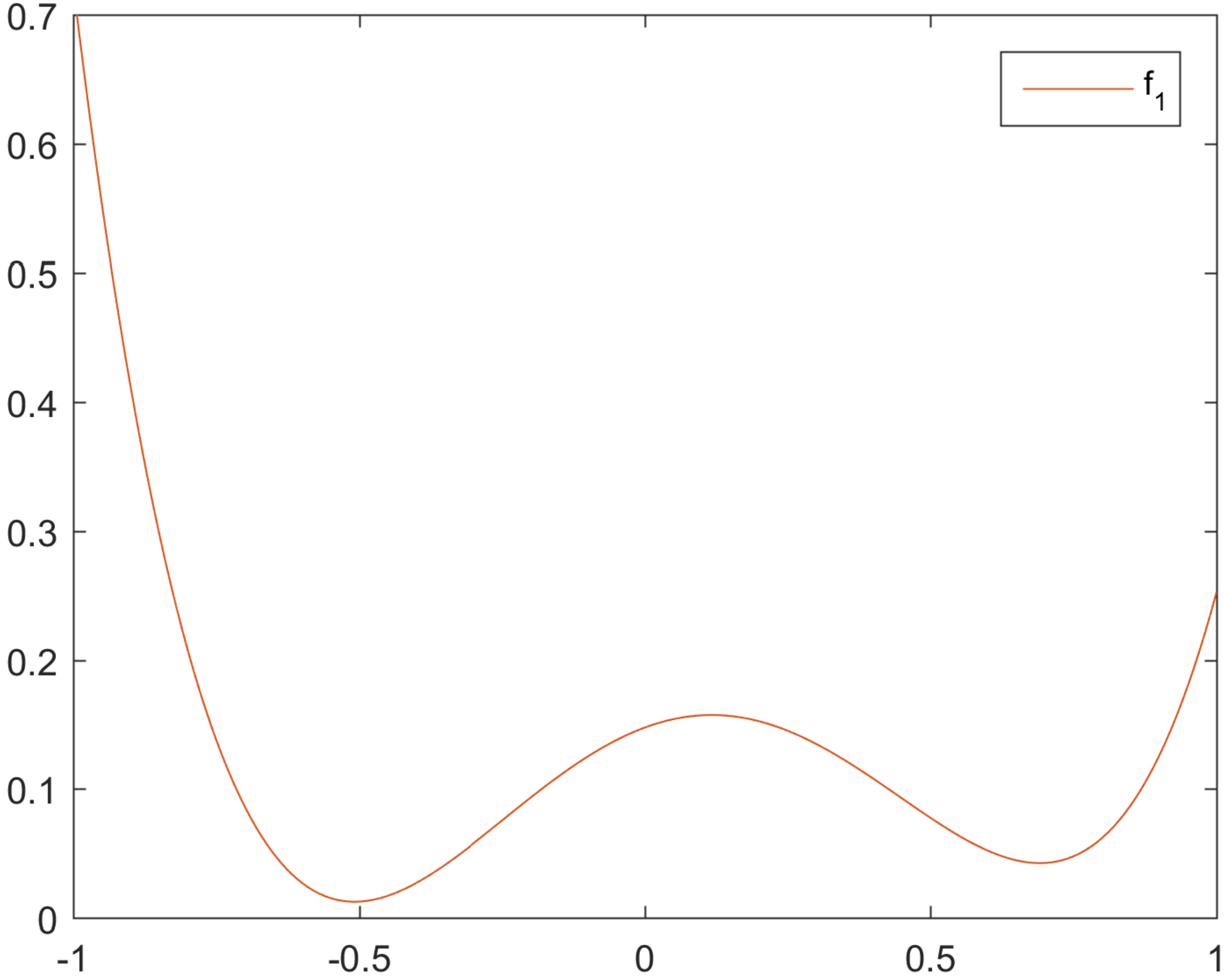

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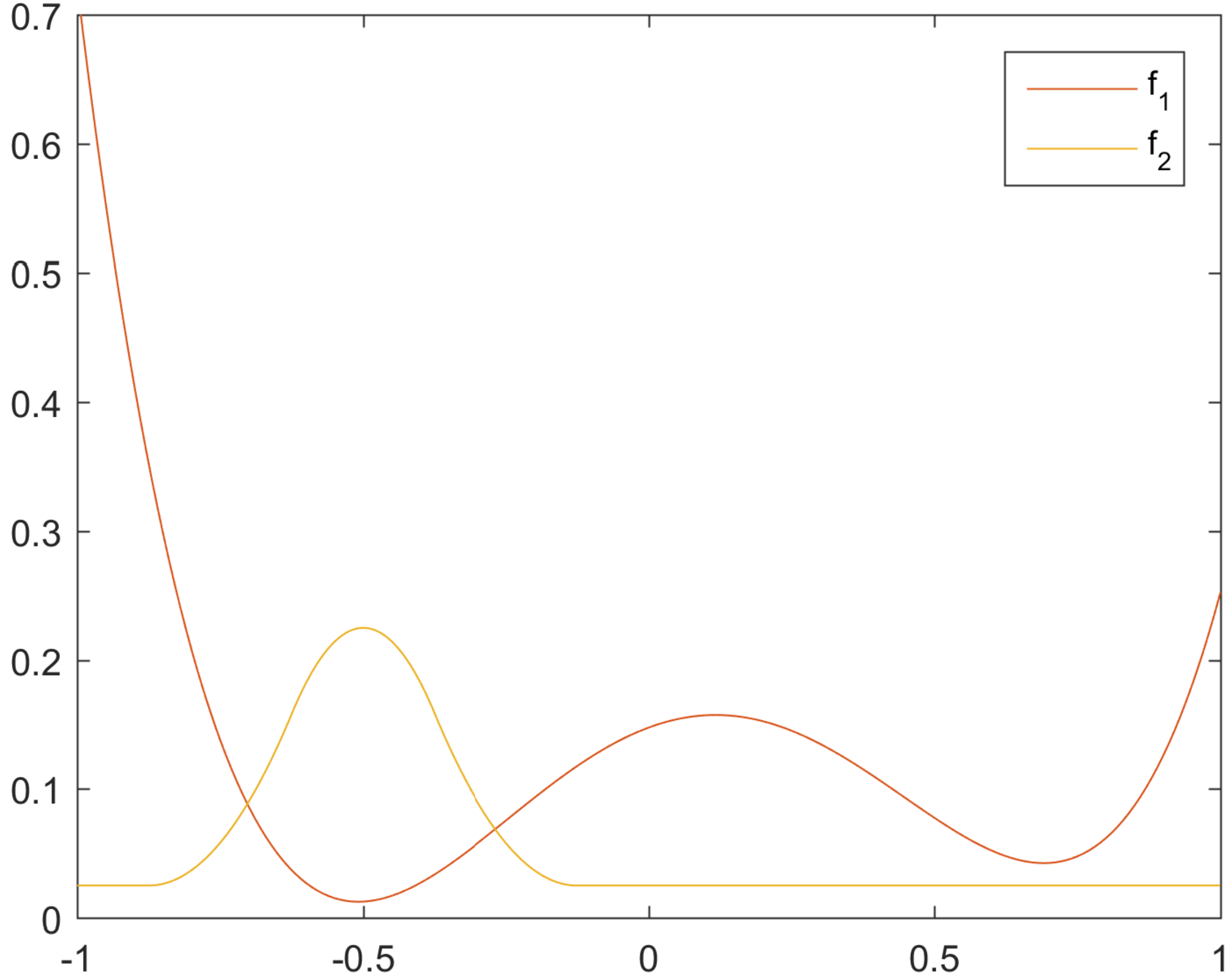

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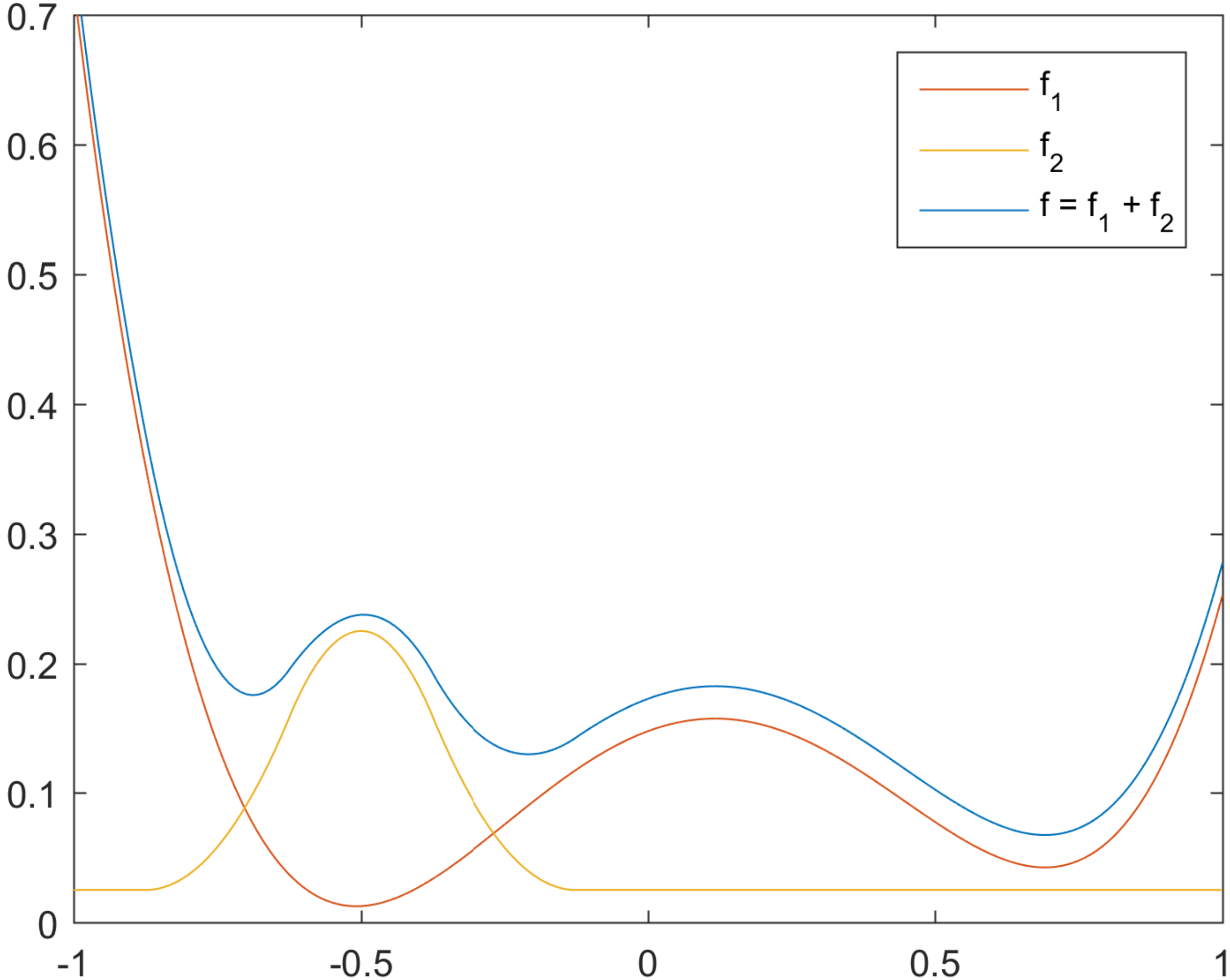

Separate exact relaxation

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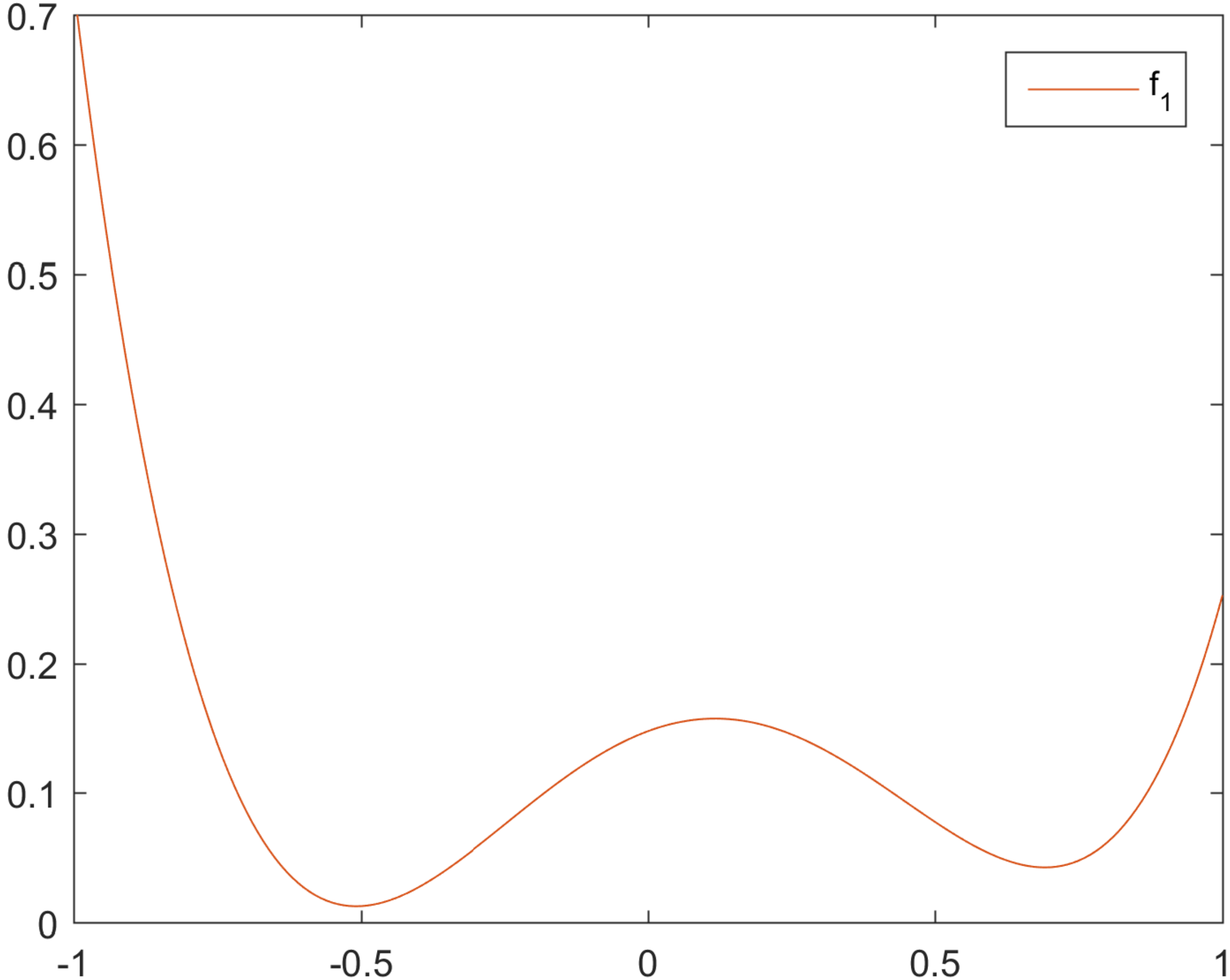

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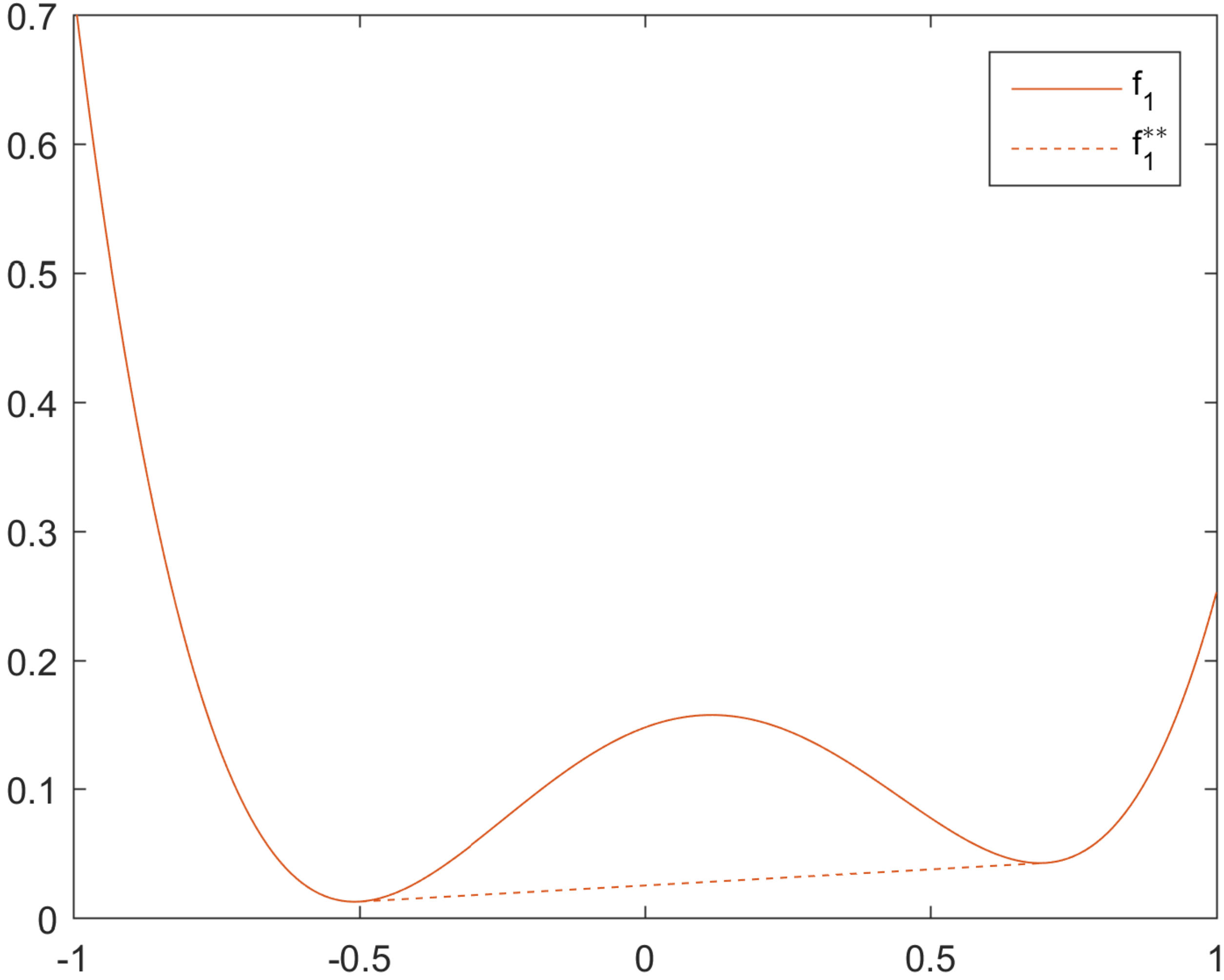

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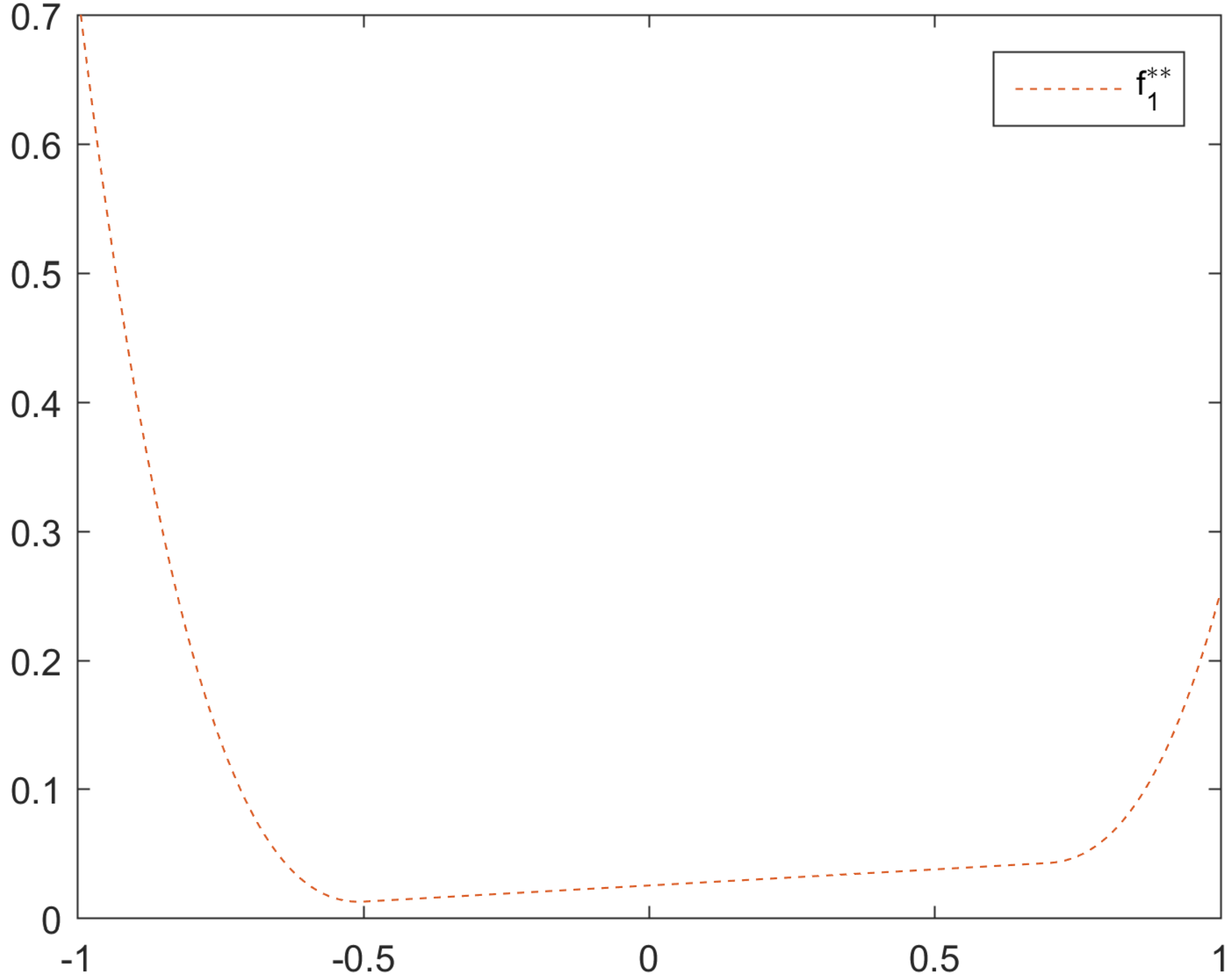

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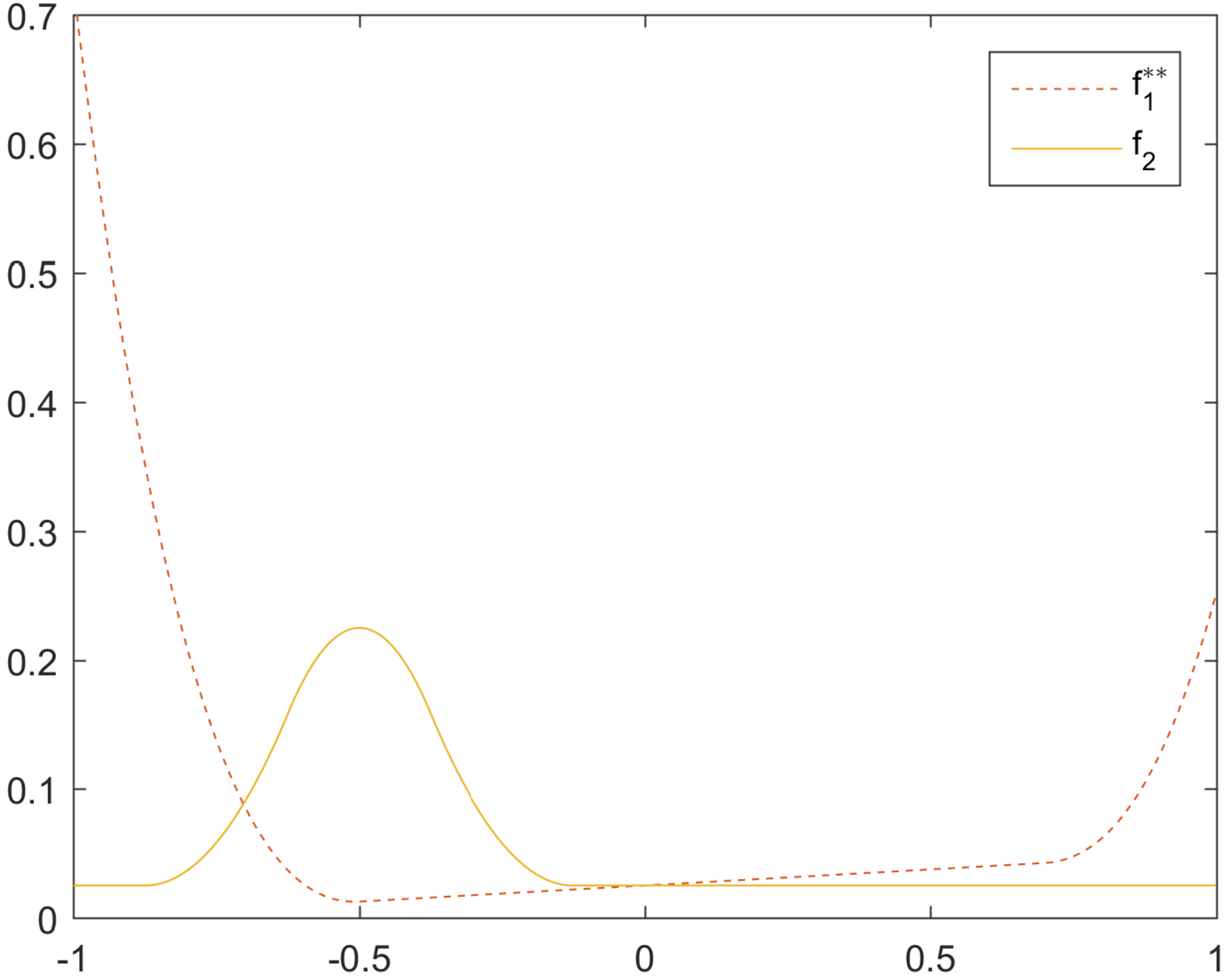

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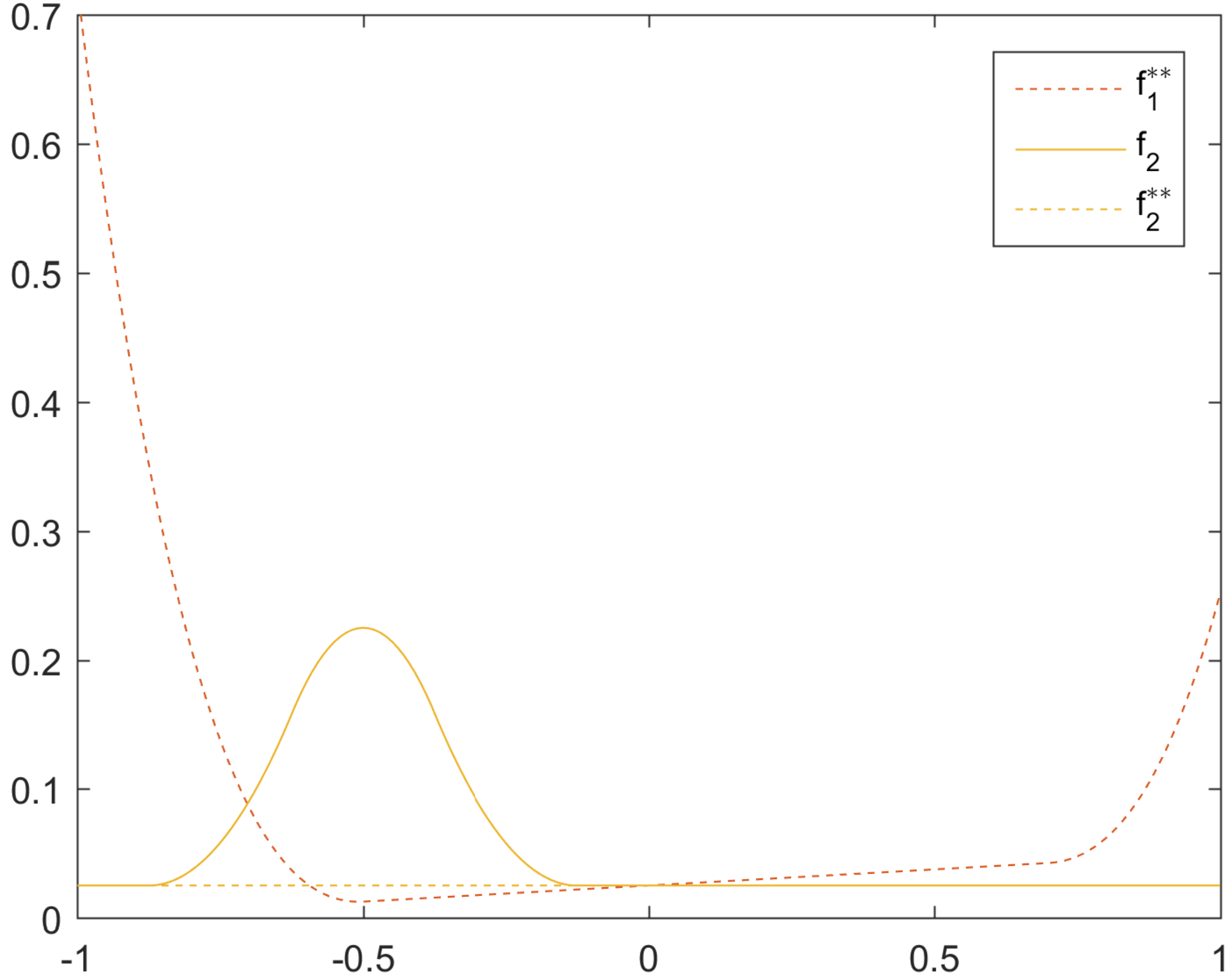

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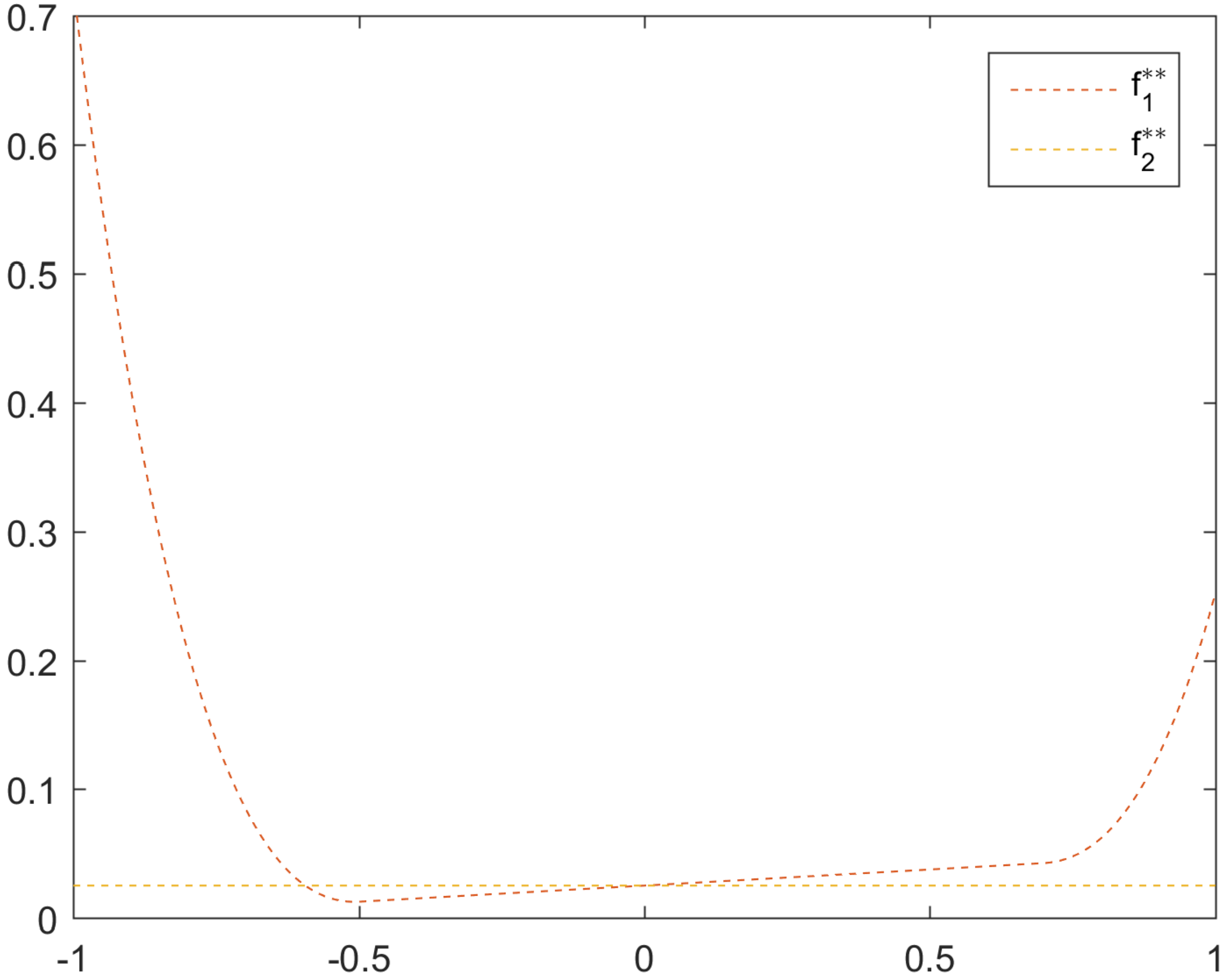

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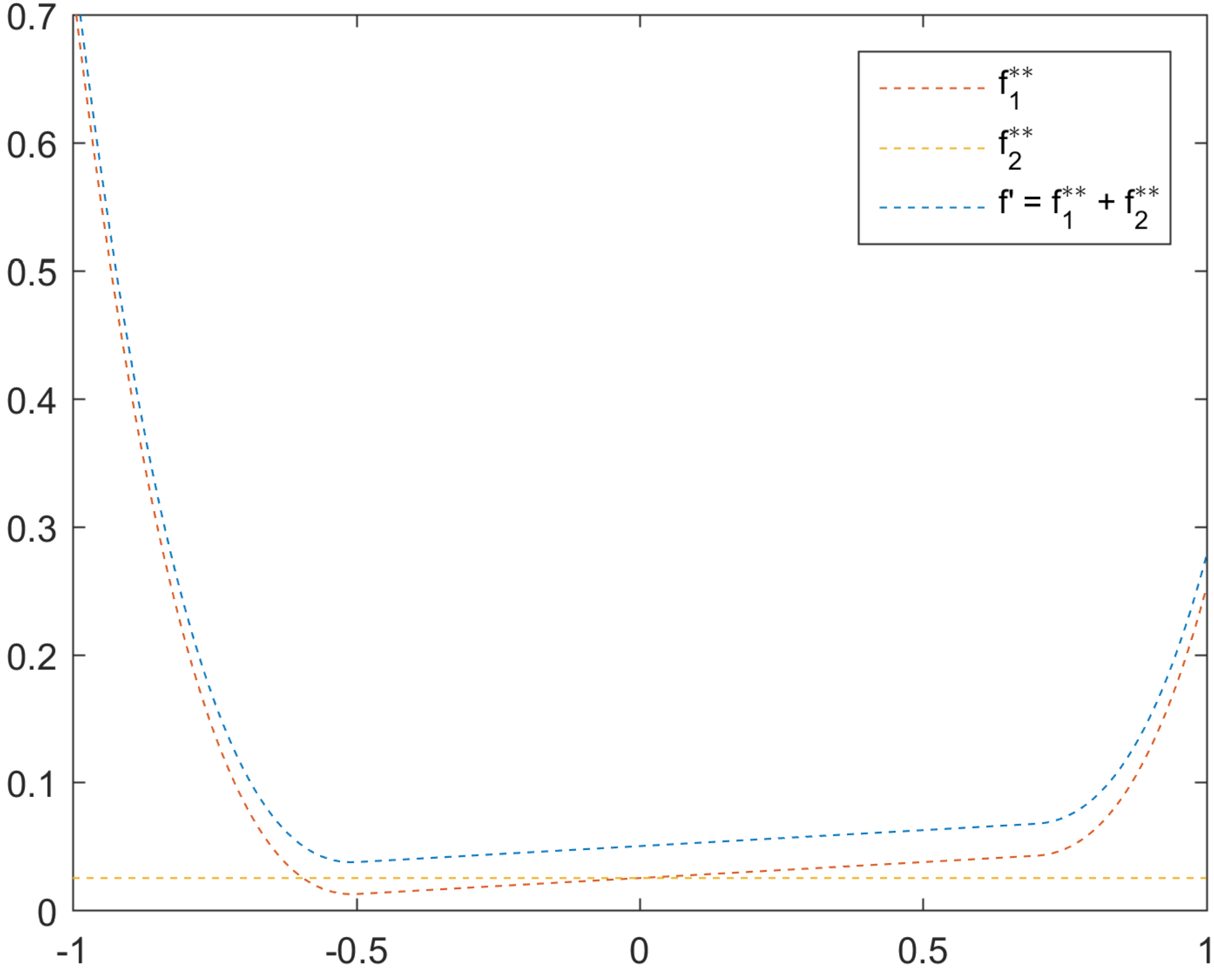

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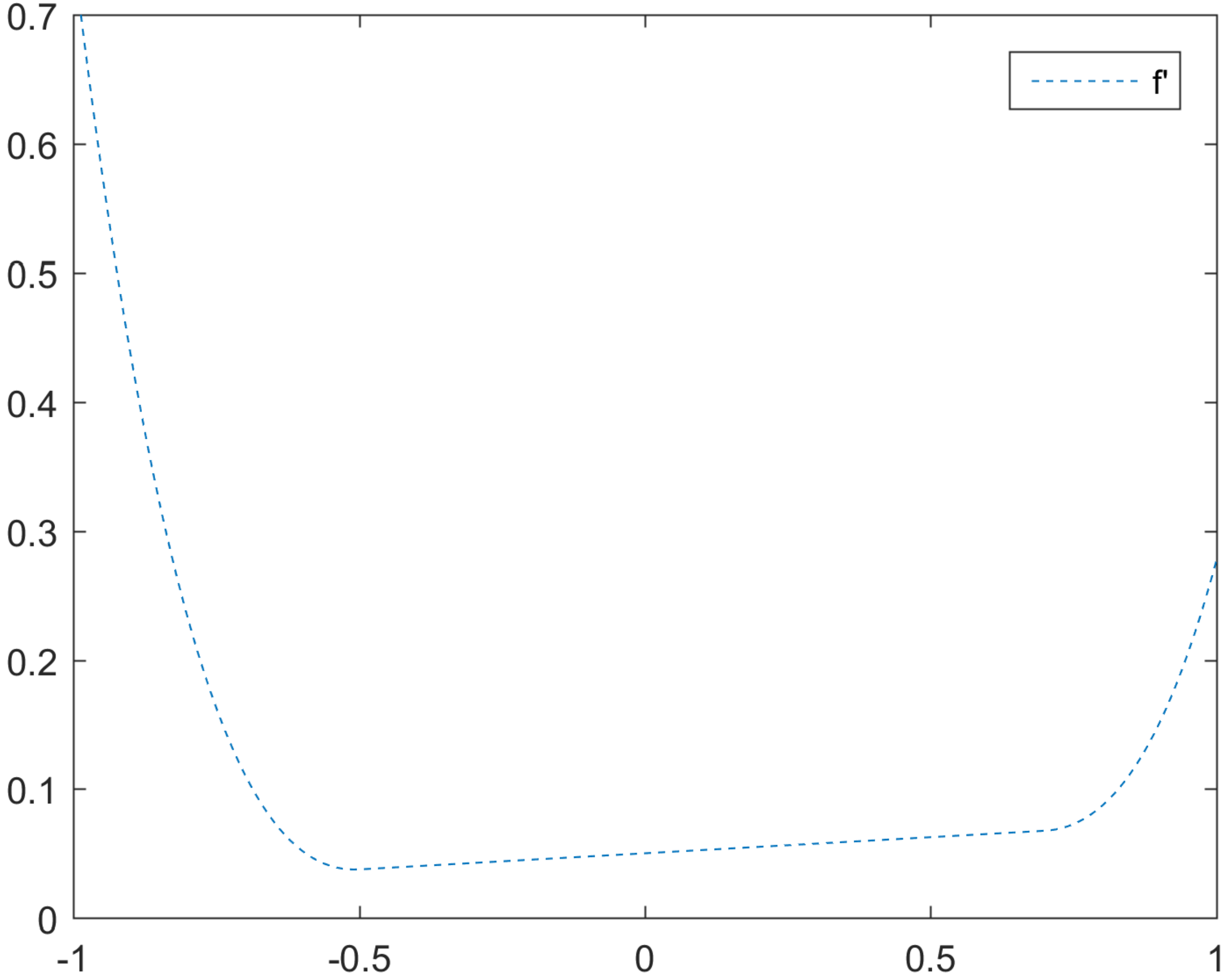

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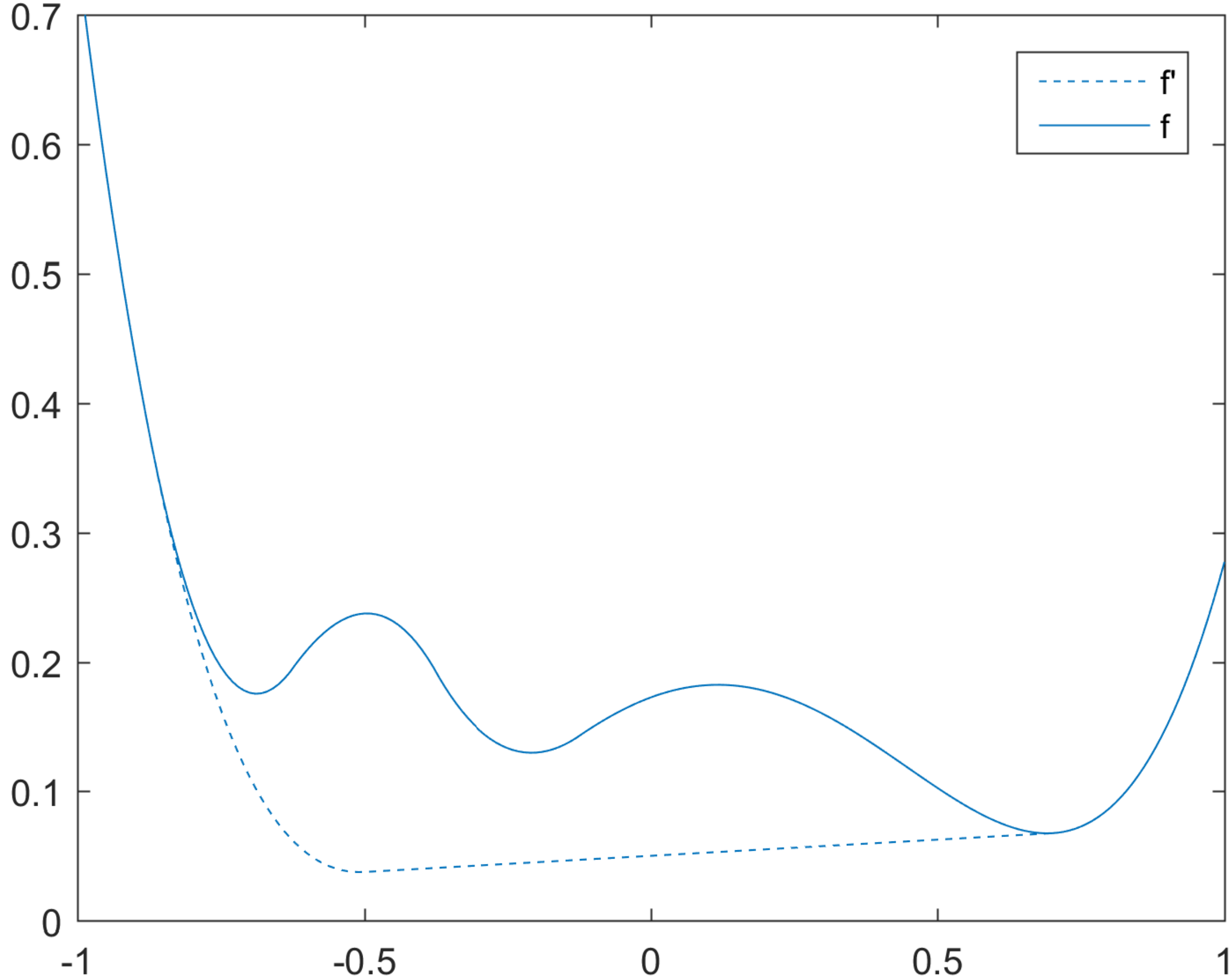

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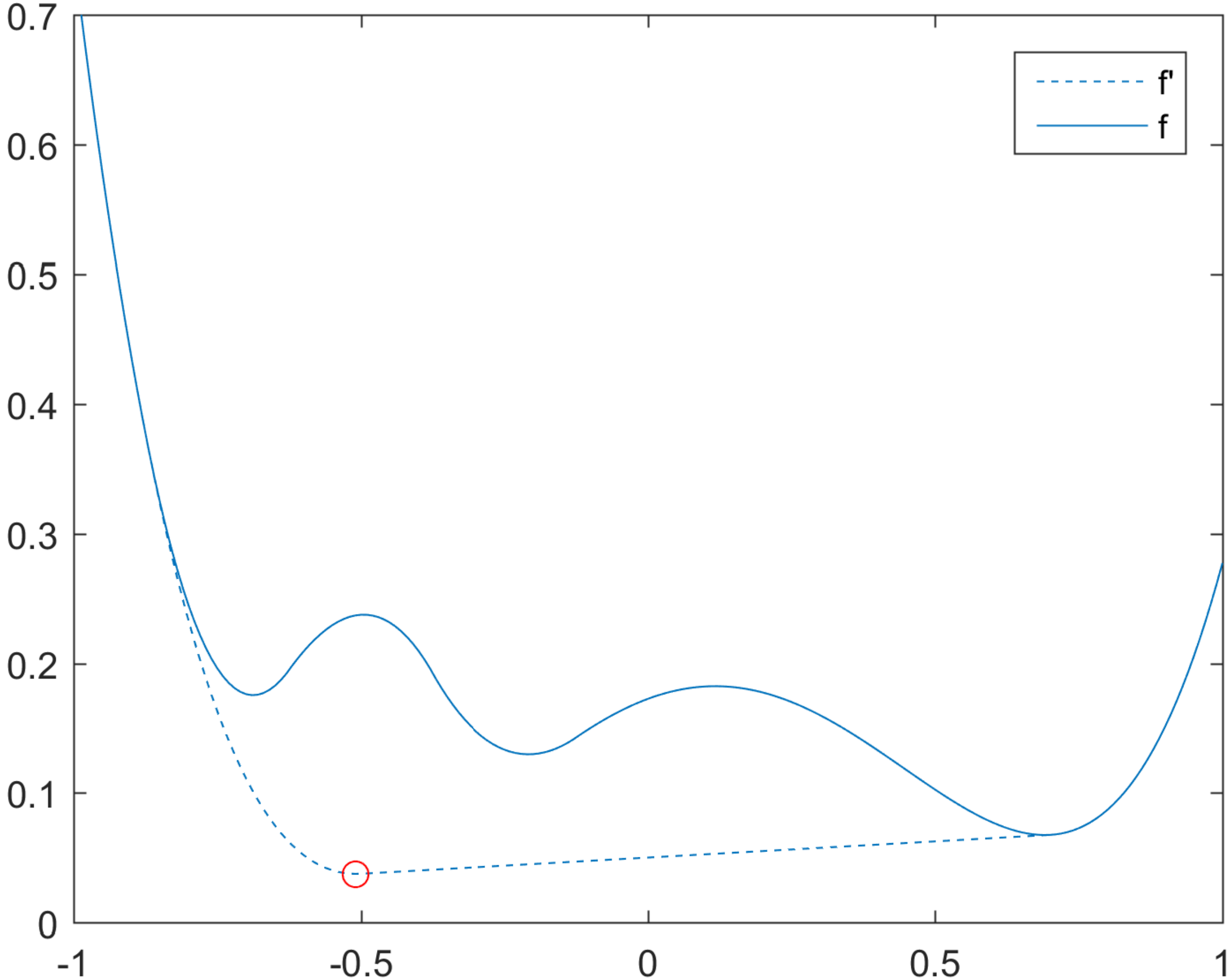
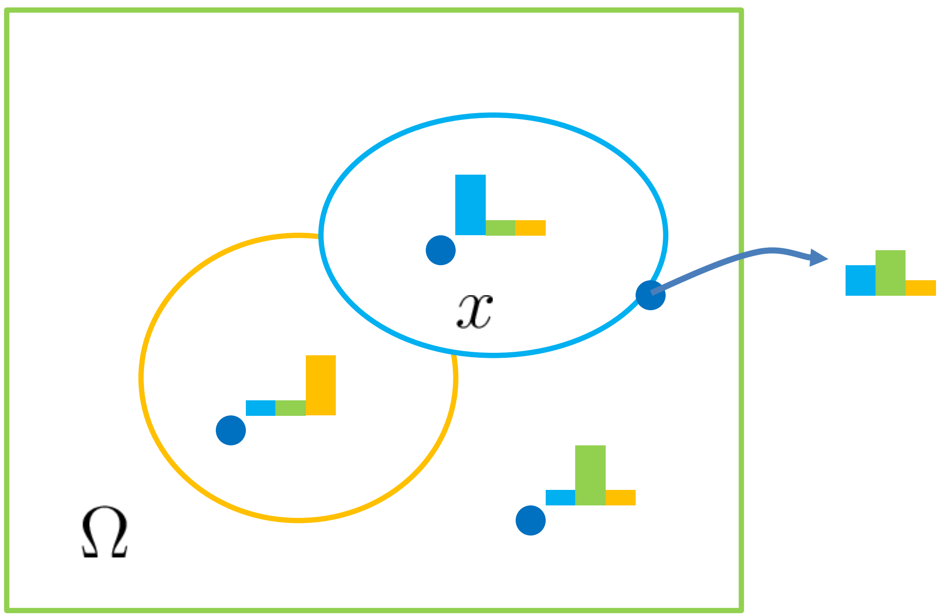


Lifting

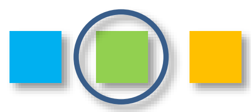


Hard decisions are replaced by soft "probabilities"

Lifting

Original problem

$$\min_{u:\Omega\to X} f(u)$$



Potts '52; Boykov et al. '98, '01; Kleinberg, Tardos '01
Zach et al. '08; Lellmann, Becker, Schnörr '09; Chambolle, Cremers, Pock '11; Yuan, Bae, Tai, Boykov'10
Young measures: Young '37; Currents: Schwartz '51; de Rahm '55; Federer '69
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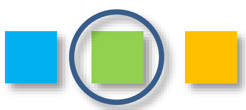
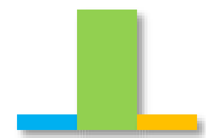


lifted to probability measures:

$$\min_{\boldsymbol{u}:\Omega\to\mathcal{P}(X)} \boldsymbol{f}(\boldsymbol{u})$$



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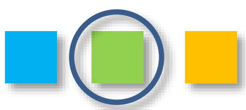

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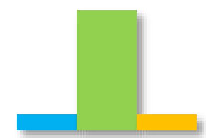

Ideally:

$$\boldsymbol{f}(\boldsymbol{u}) = \begin{cases} f(u), & \text{if } \ \boldsymbol{u}(x) = \delta_{u(x)} \ \ \forall x \\ +\infty, & \text{otherwise} \end{cases}$$

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Linear relaxation

- Lifting + relaxation using the biconjugate:

$$\int_\Omega \rho(u(x))dx \rightsquigarrow \int_\Omega \boldsymbol{\rho}^{**}(\boldsymbol{u}(x))dx$$

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$$\boldsymbol{\rho}(z) = \begin{cases} \rho(t^i), & z = e^i,\ i \in \{1, \dots, L\}, \\ +\infty, & \text{otherwise.} \end{cases}$$

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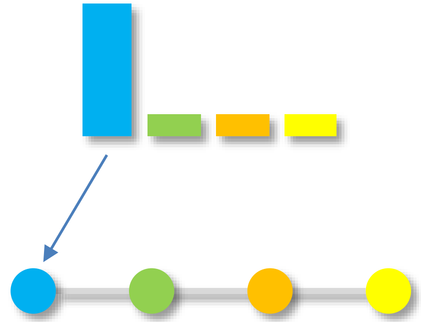

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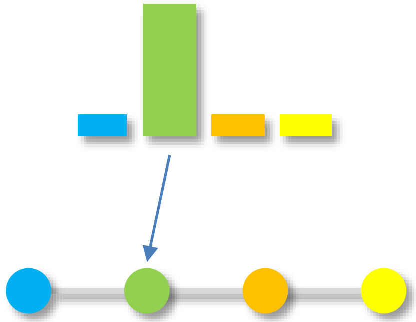

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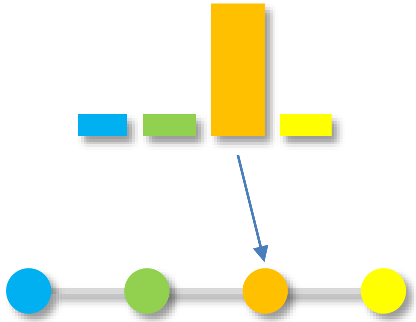

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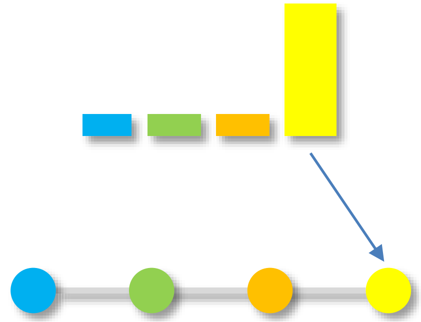

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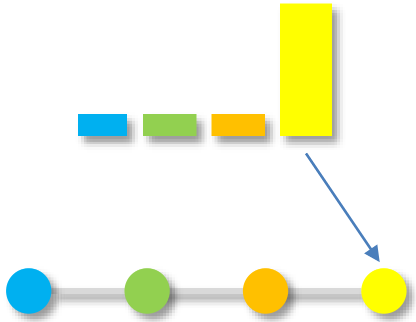

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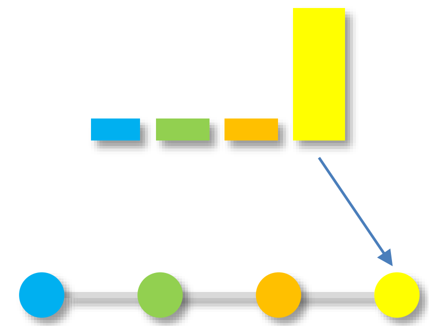

$$\min_{\boldsymbol{u} \in \mathrm{BV}(\Omega, \mathcal{P}(X))} \boldsymbol{f}(\boldsymbol{u}) := \int_\Omega \langle \boldsymbol{u}(x), s(x) \rangle dx + \int_\Omega d\Psi(D\boldsymbol{u})$$

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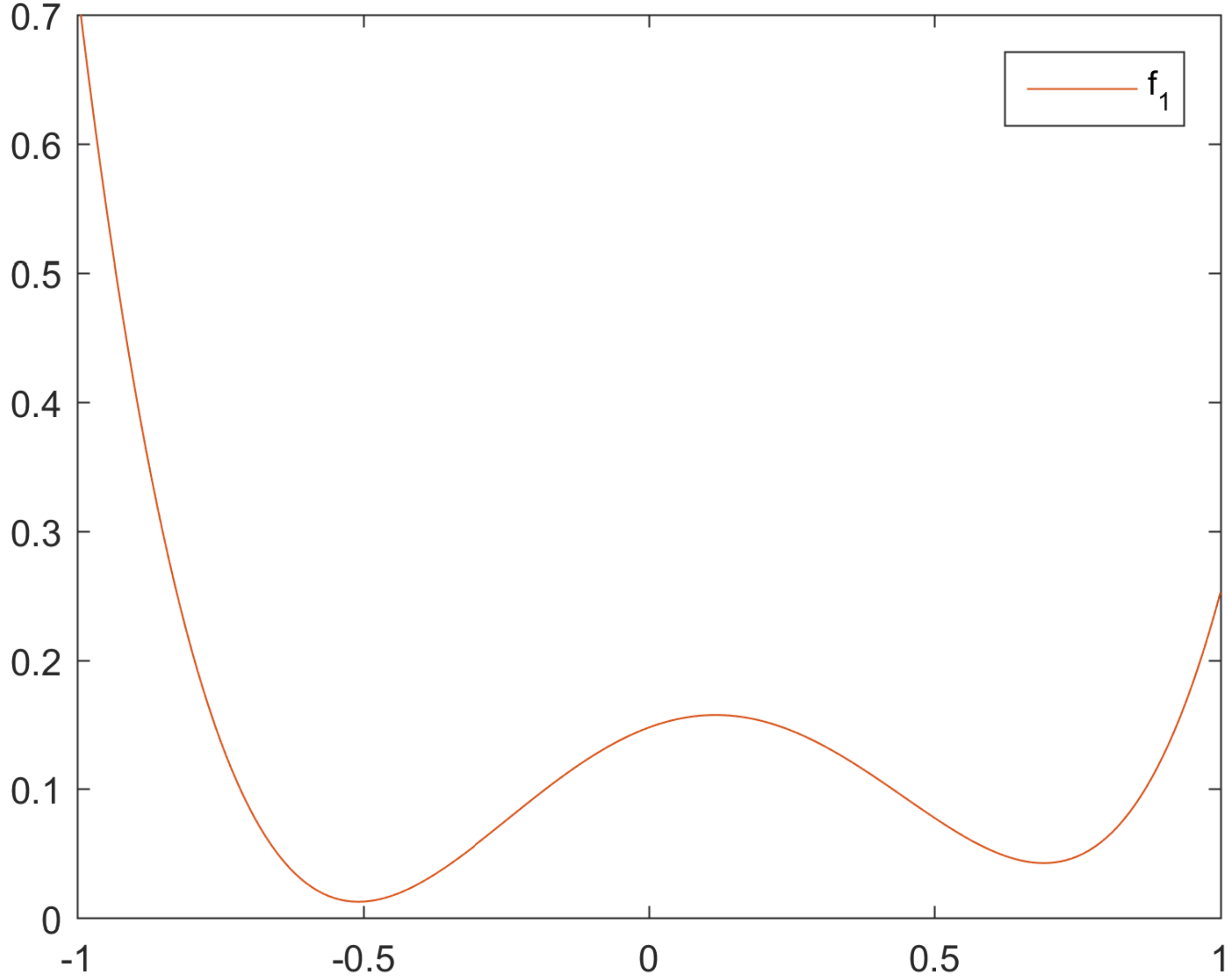

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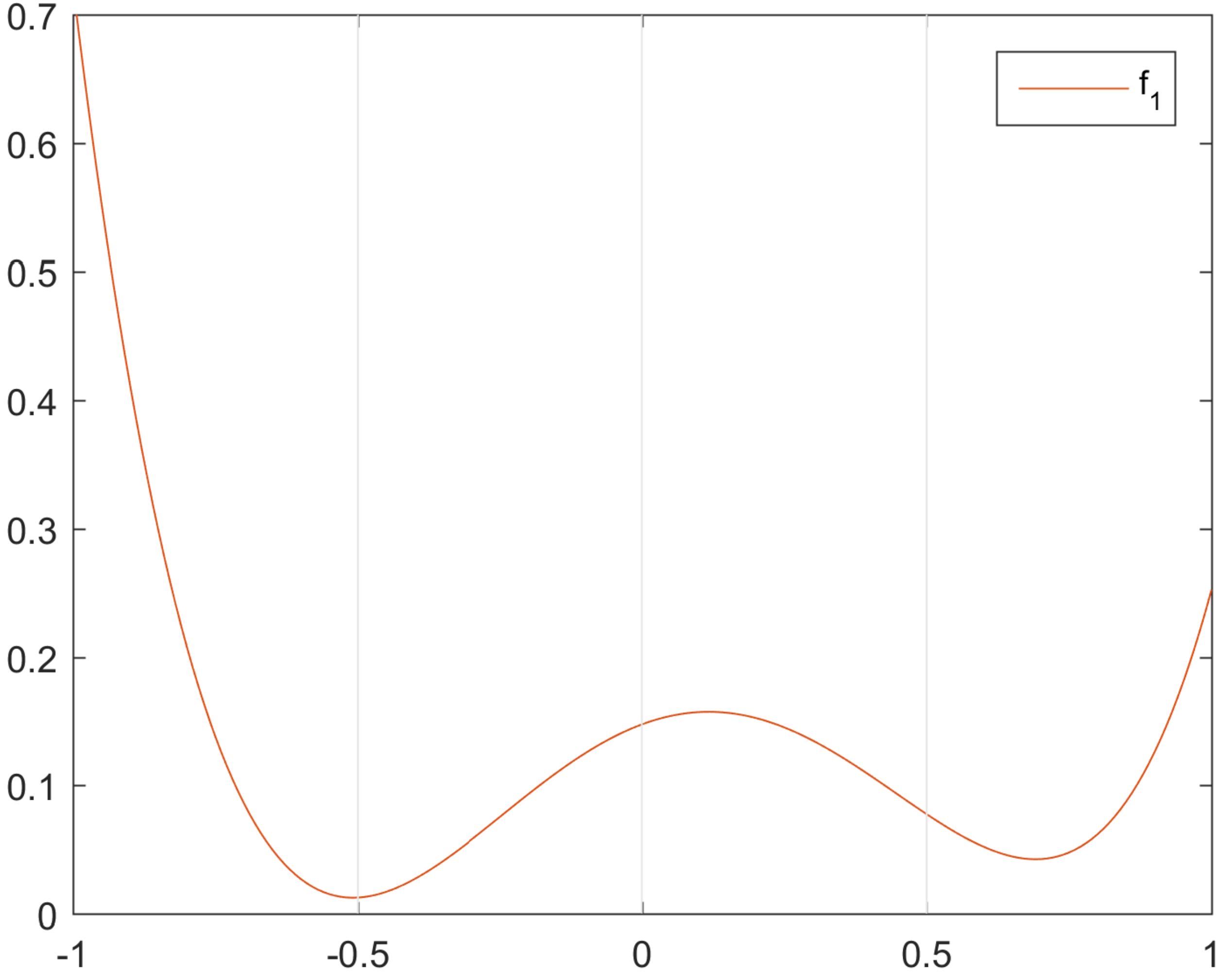

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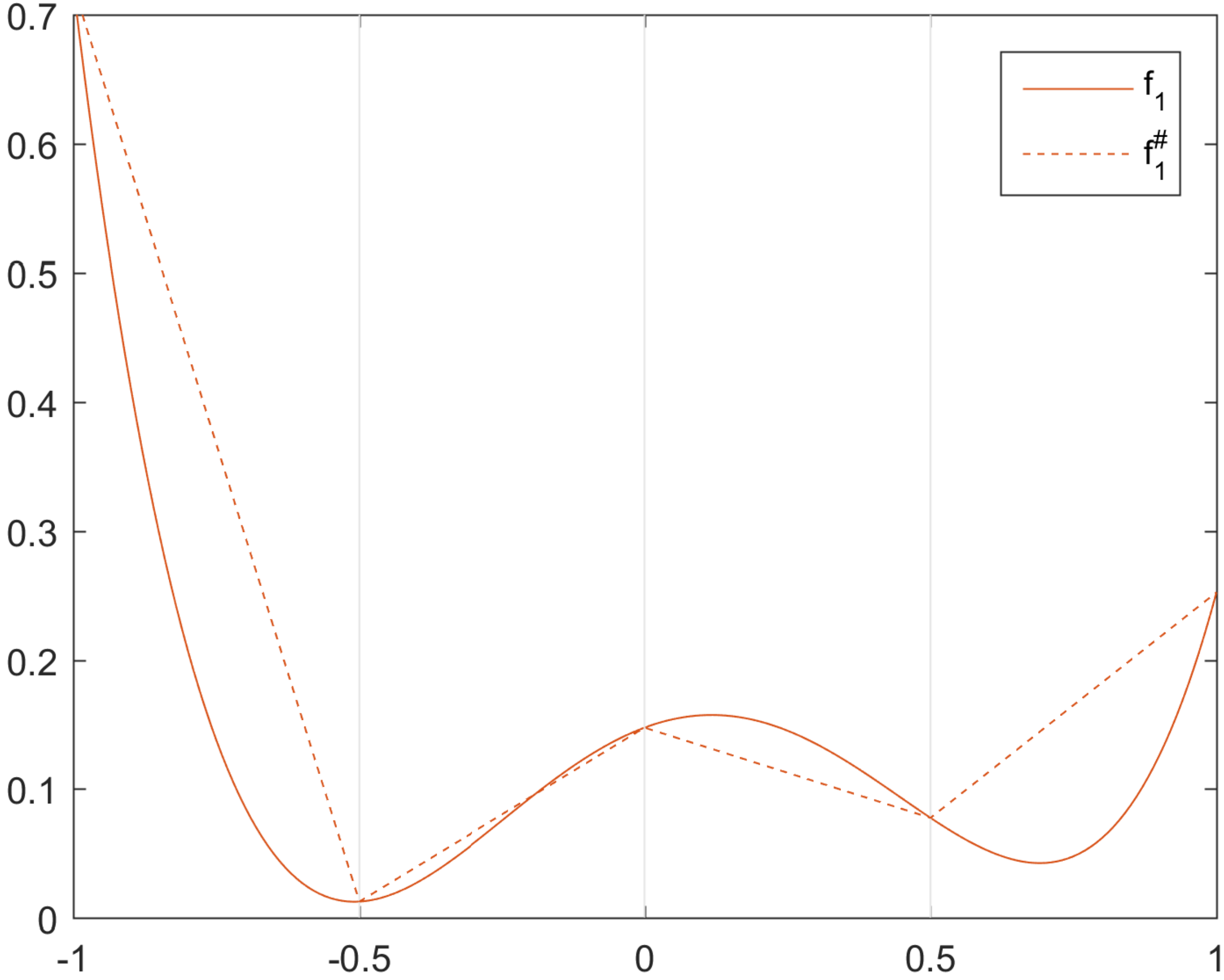

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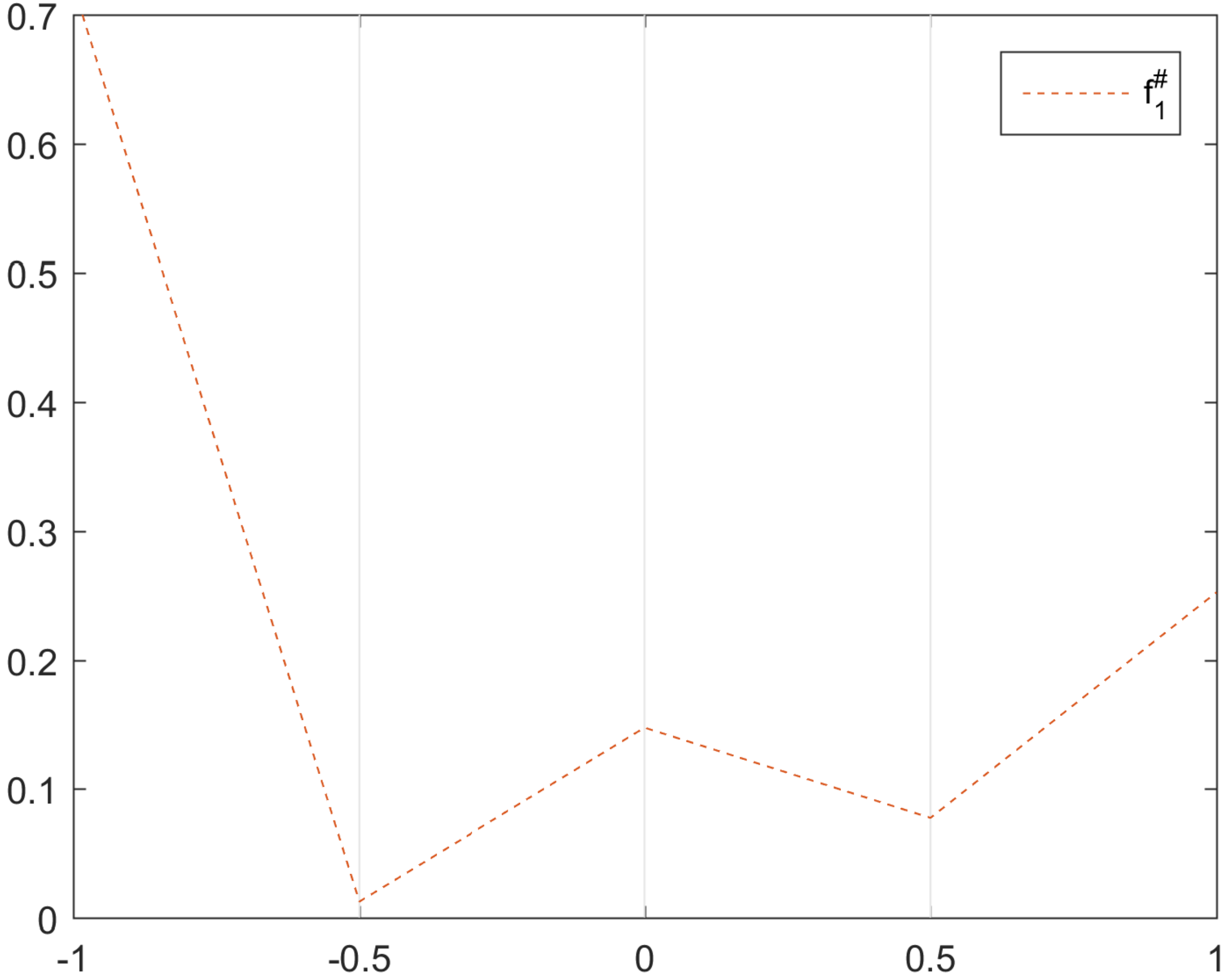

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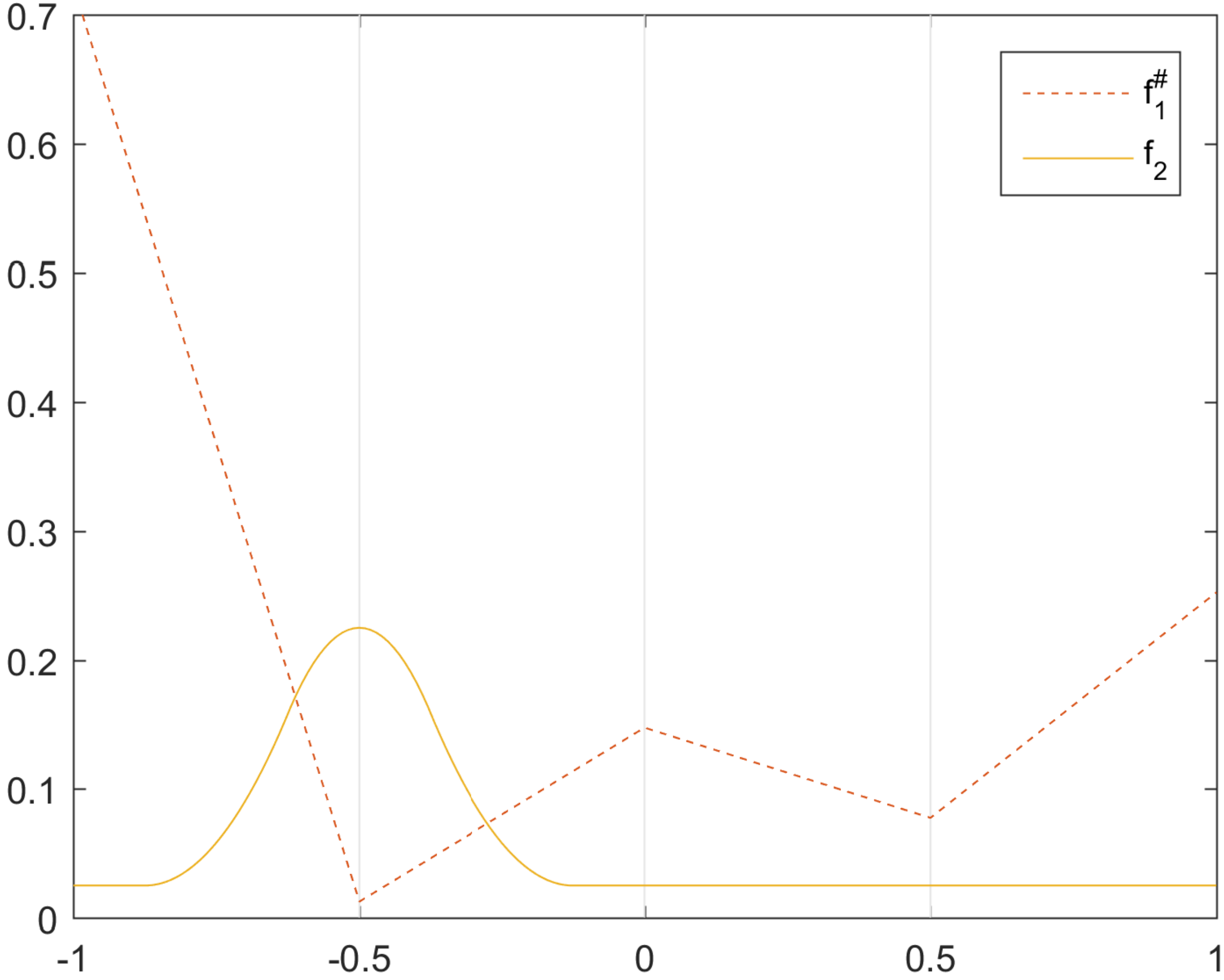

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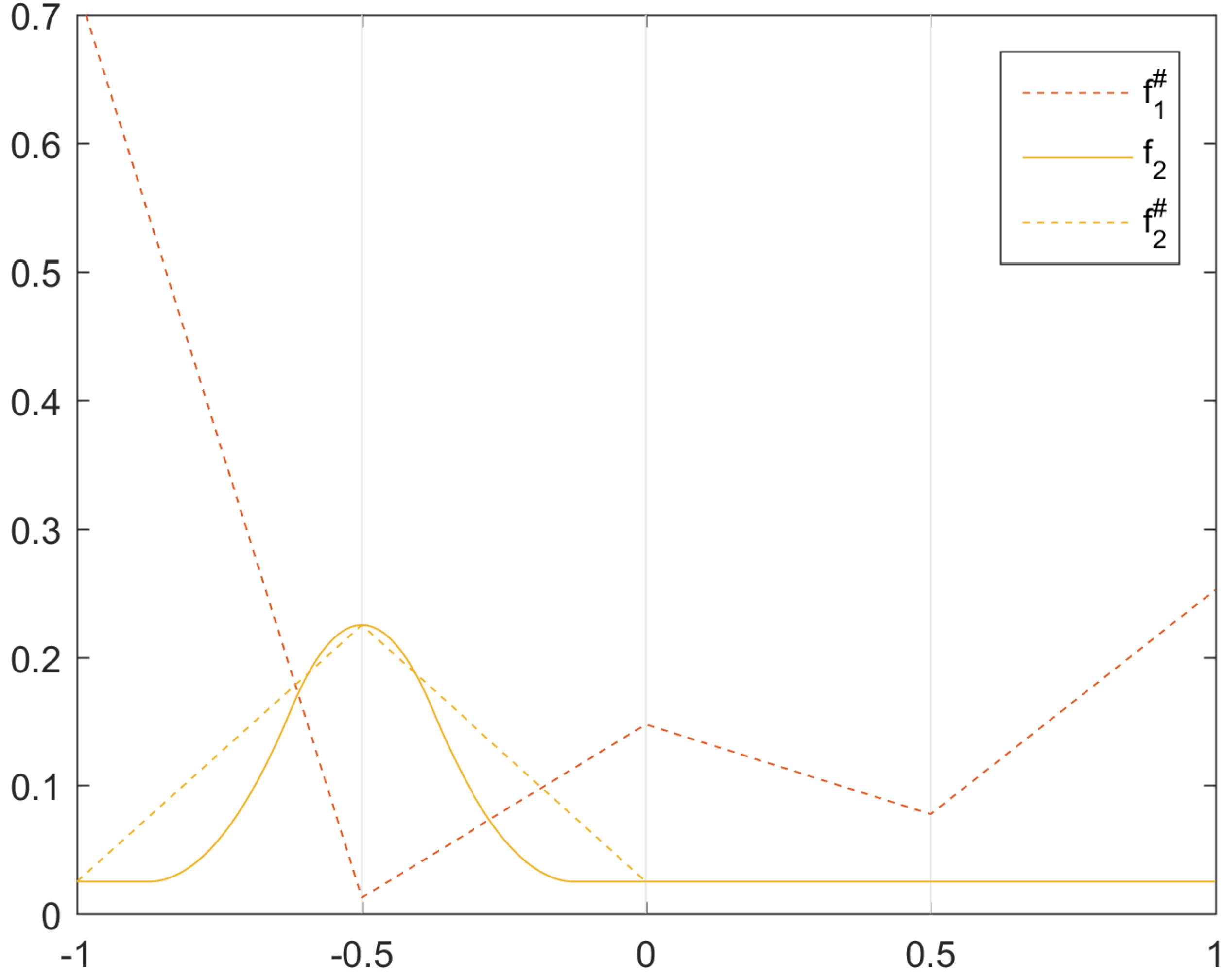

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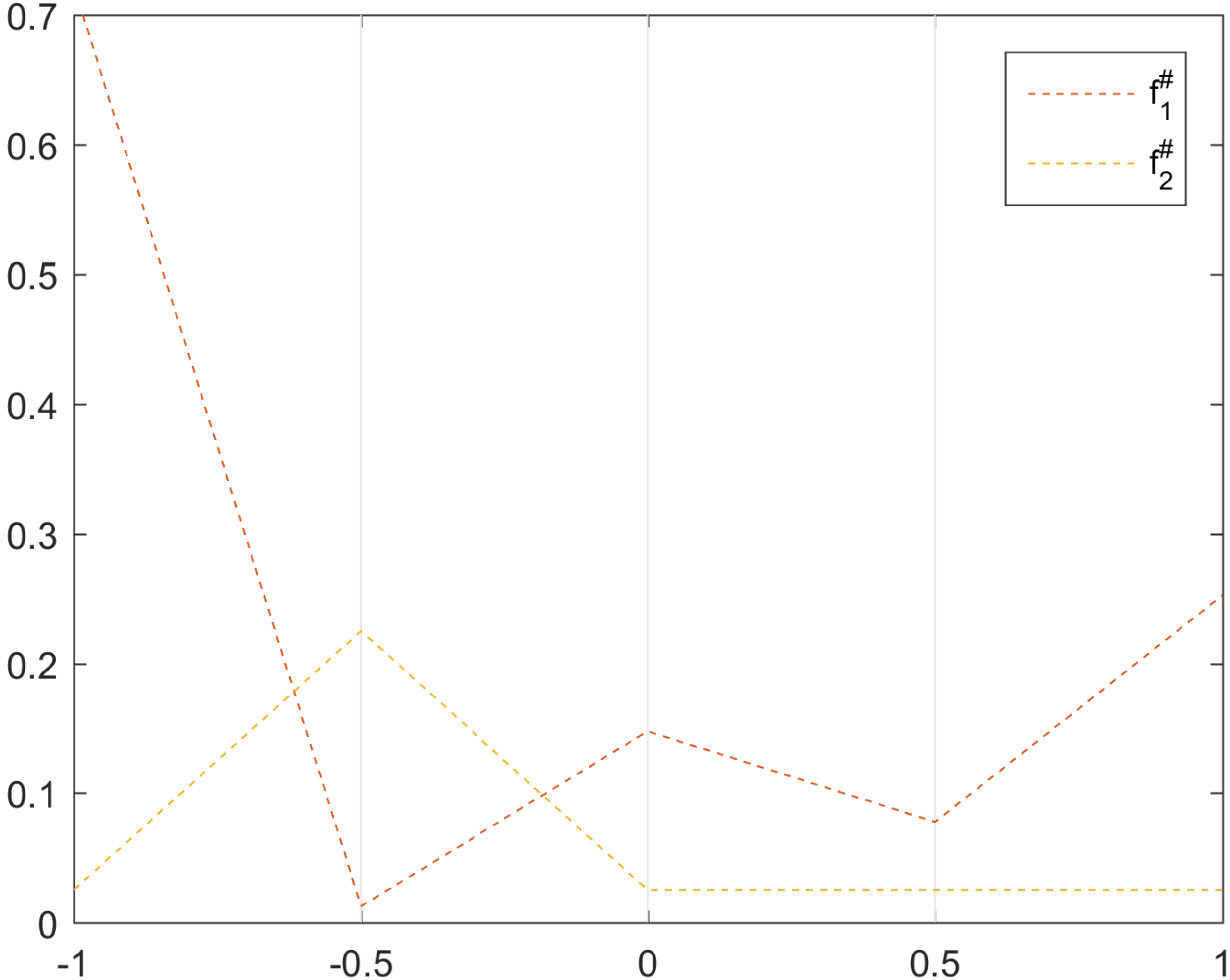

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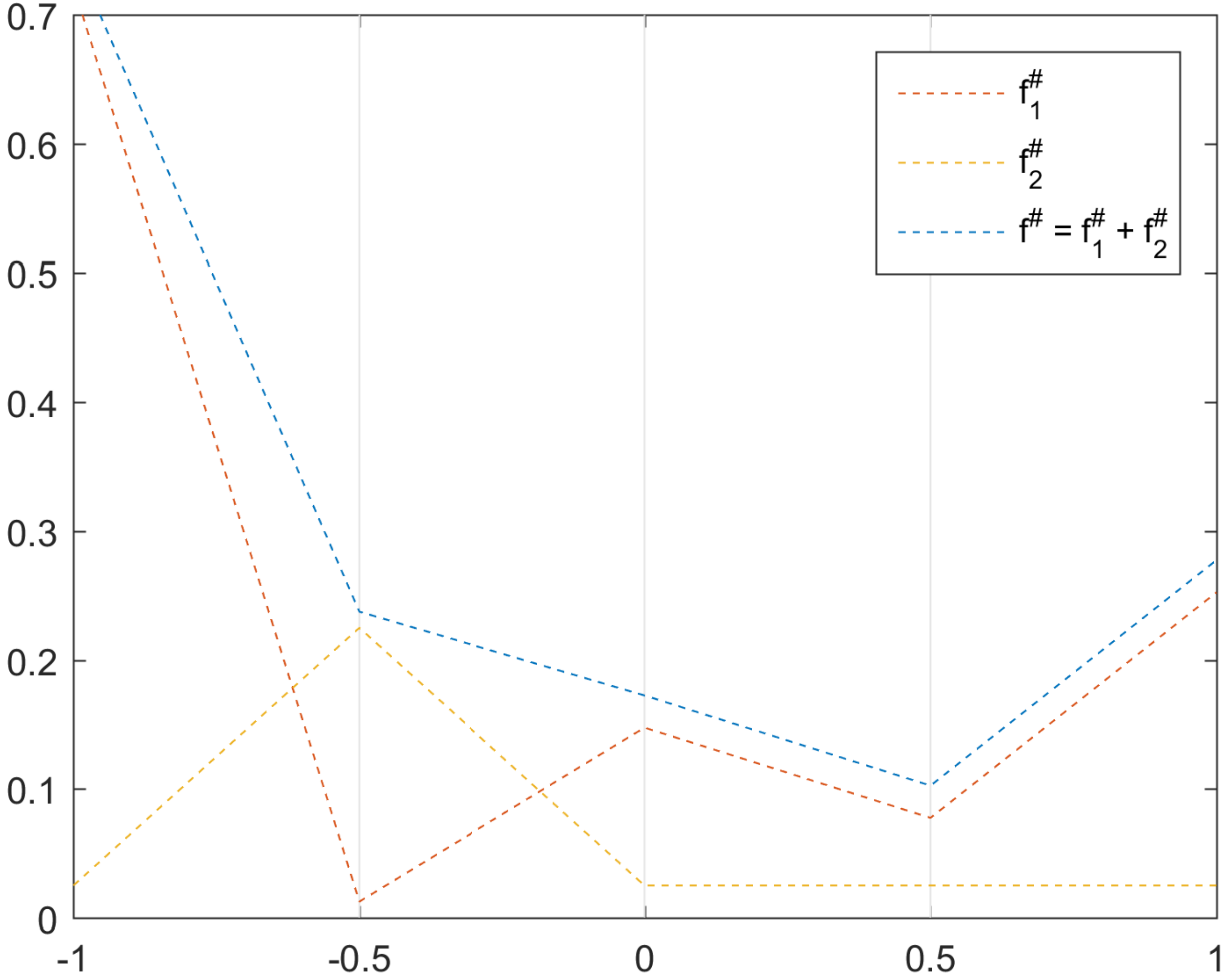

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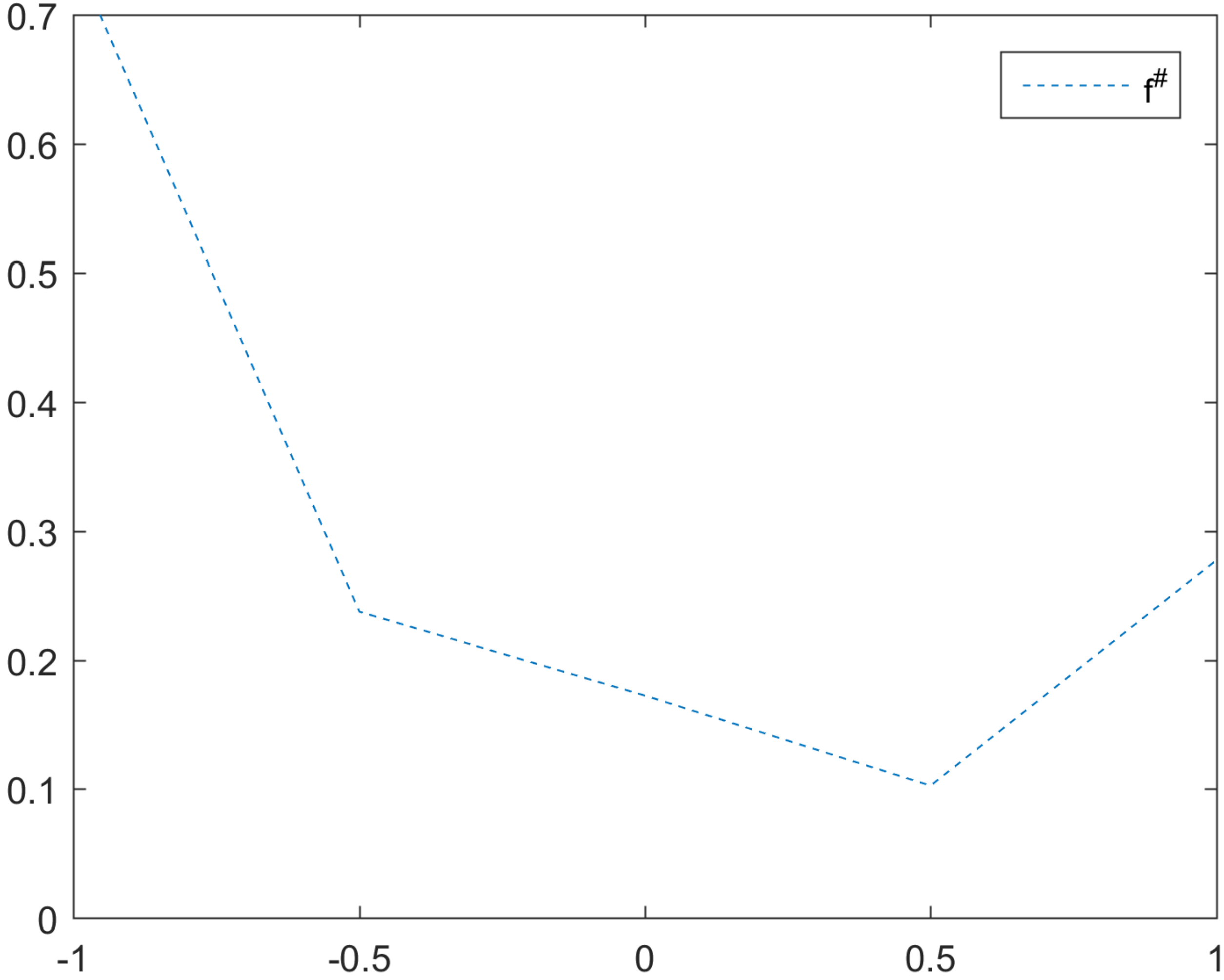

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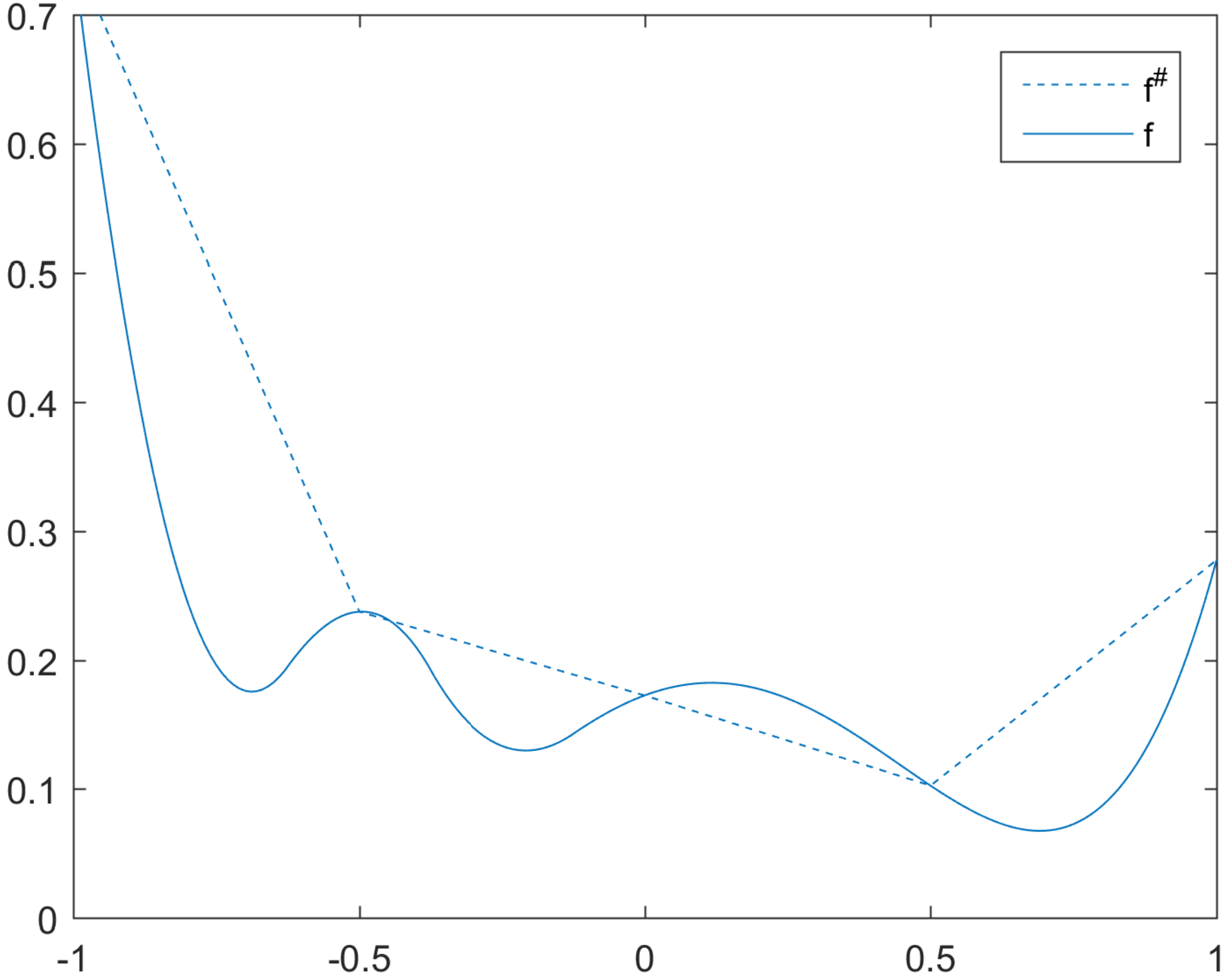

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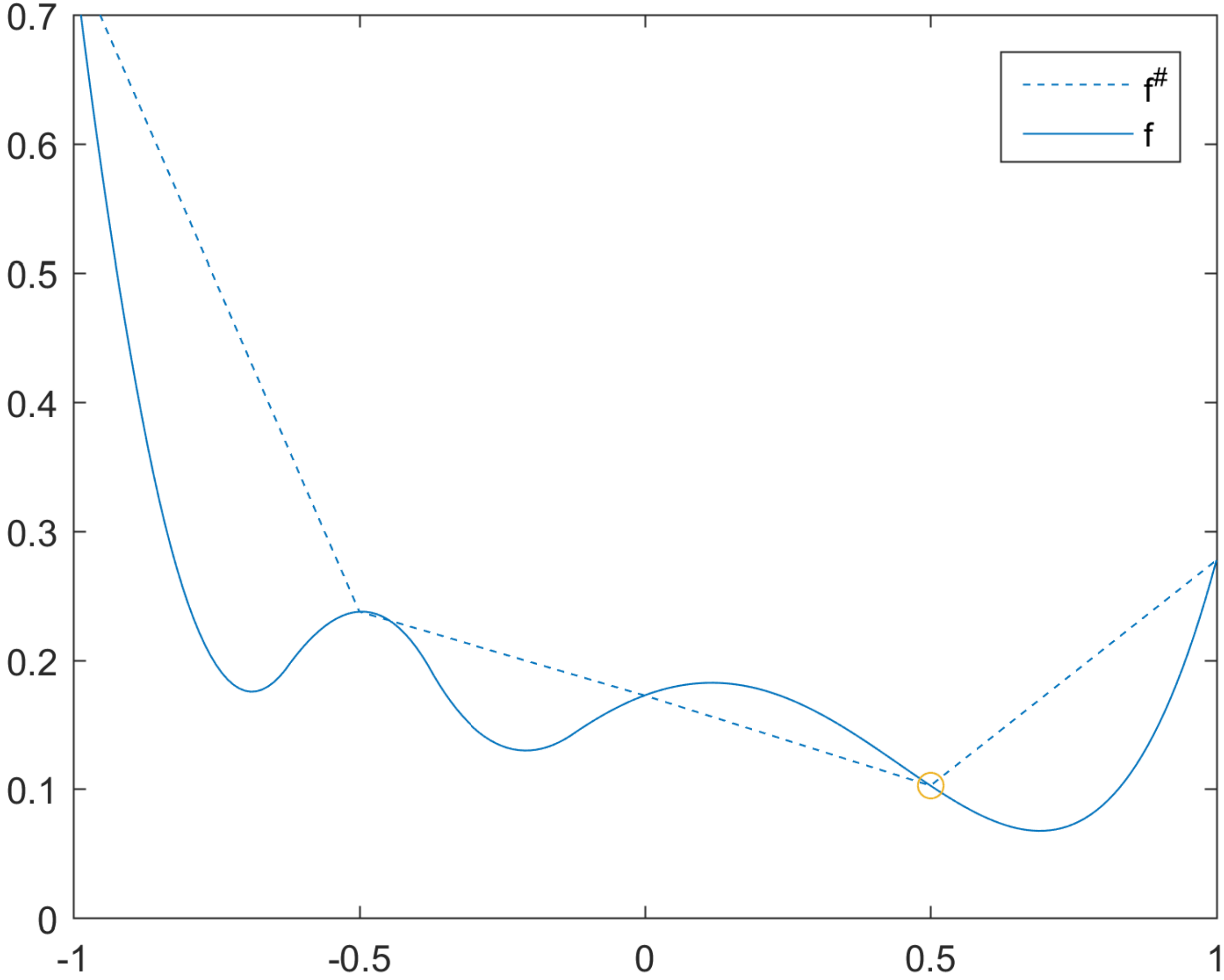
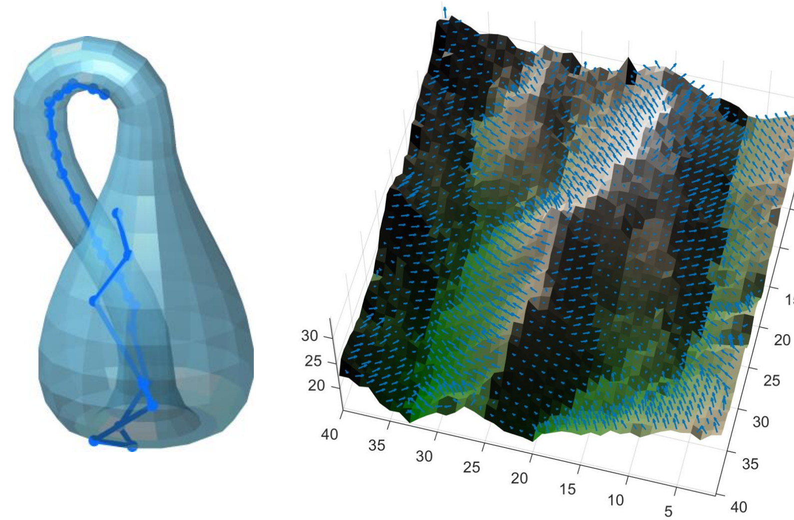
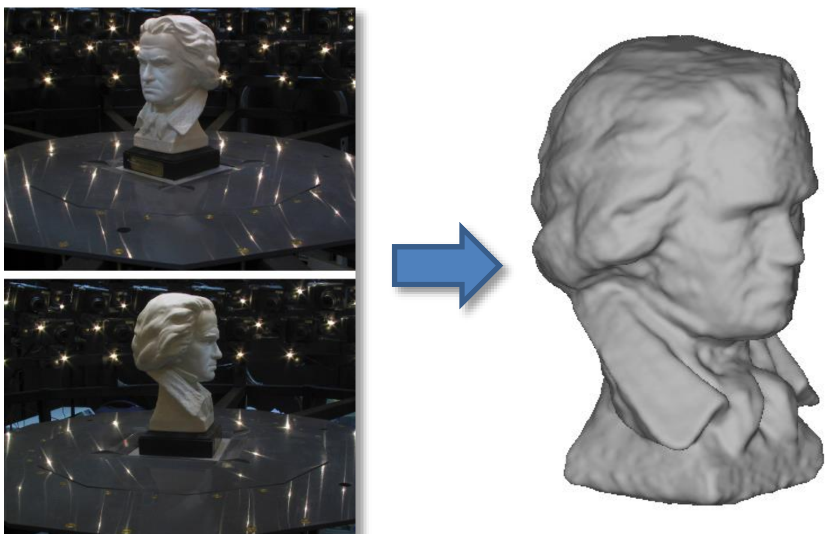


RGB-depth segmentation
(Diebold et al. , SSVM '15)



Restoring manifold-valued data
(Cremers, Strekalovskiy '12,
Lellmann et al., ICCV'13)
related: Weinmann, Demart, Storath'14;
Bergman et al.'14



3D reconstruction
(Kolev et al., Int. J. Comp. Vis. '09)

Is it optimal?

Algorithm 1 Continuous Probabilistic Rounding

$u^0 \leftarrow u$, $U^0 \leftarrow \Omega$, $c^0 \leftarrow (1,\ldots,1) \in \mathbb{R}^l$
for $k = 1, 2, \ldots$ **do**
Randomly choose $\gamma^k = (i^k, \alpha^k) \in \mathcal{I}$
$M^k \leftarrow U^{k-1} \cap \{x \in \Omega | u^{k-1}(x) > \alpha$

e

Proposition 2 *For the sequences (*
by Alg. 1, define

$$A := \bigcap_{k=1}^{\infty} \{\gamma \in \Gamma | \operatorname{Per}(U^k_\gamma) < \infty\}.$$

Then

$$\mathbb{P}(A) = 1.$$

If $\operatorname{Per}(U^k_\gamma) < \infty$ *for all* k, *then* u^k_γ *as well. Moreover,*

$$\mathbb{P}(u^k \in \mathrm{BV}(\Omega)^l \wedge \operatorname{Per}(U^k) < \infty \forall k$$

i.e., the algorithm almost surely generates a sequence of BV *functions* (u^k) *and a sequence of sets of finite perimeter* (U^k).

Theorem 1 *Let* $u \in \mathcal{C}$, $s \in L^1(\Omega)^l$, $s \geqslant 0$, *and let* $\Psi : \mathbb{R}^{d \times l} \to \mathbb{R}_{\geqslant 0}$ *be positively homogeneous, convex and continuous. Assume there exists a lower bound* $\lambda_l > 0$ *such that, for* $z = (z^1, \ldots, z^l)$,

$$\Psi(z) \geqslant \lambda_l \frac{1}{2} \sum_{i=1}^{l} \|z^i\|_2 \quad \forall z \in \mathbb{R}^{d \times l}, \sum_{i=1}^{l} z^i = 0. \tag{24}$$

Moreover, assume there exists an upper bound $\lambda_u < \infty$ *such that, for every* $\nu \in \mathbb{R}^d$ *satisfying* $\|\nu\|_2 = 1$,

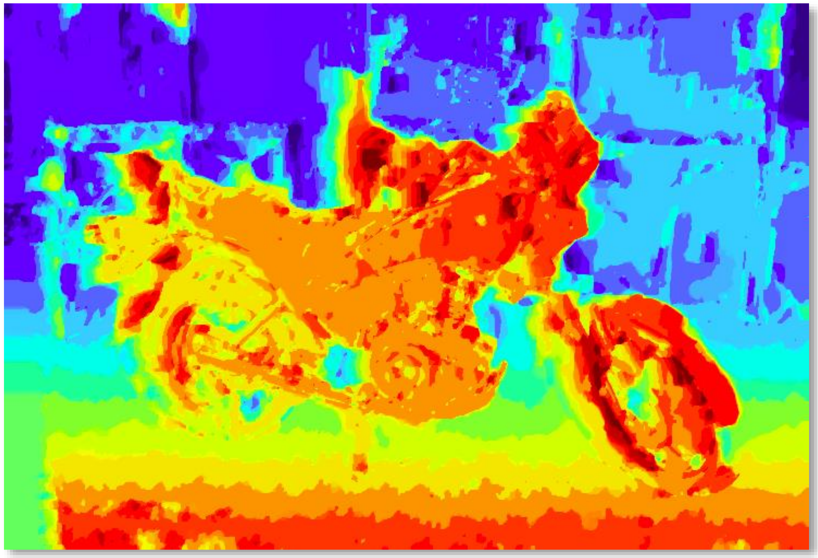
$$\Psi(\nu(e^i - e^j)^\top) \leqslant \lambda_u \quad \forall i, j \in \{1, \ldots, l\}. \tag{25}$$

Then Alg. 1 (see below) generates an integral labeling $\bar{u} \in \mathcal{C}_\mathcal{E}$ *almost surely, and*

$$\mathbb{E} f(\bar{u}) \leqslant 2 \frac{\lambda_u}{\lambda_l} f(u). \tag{26}$$

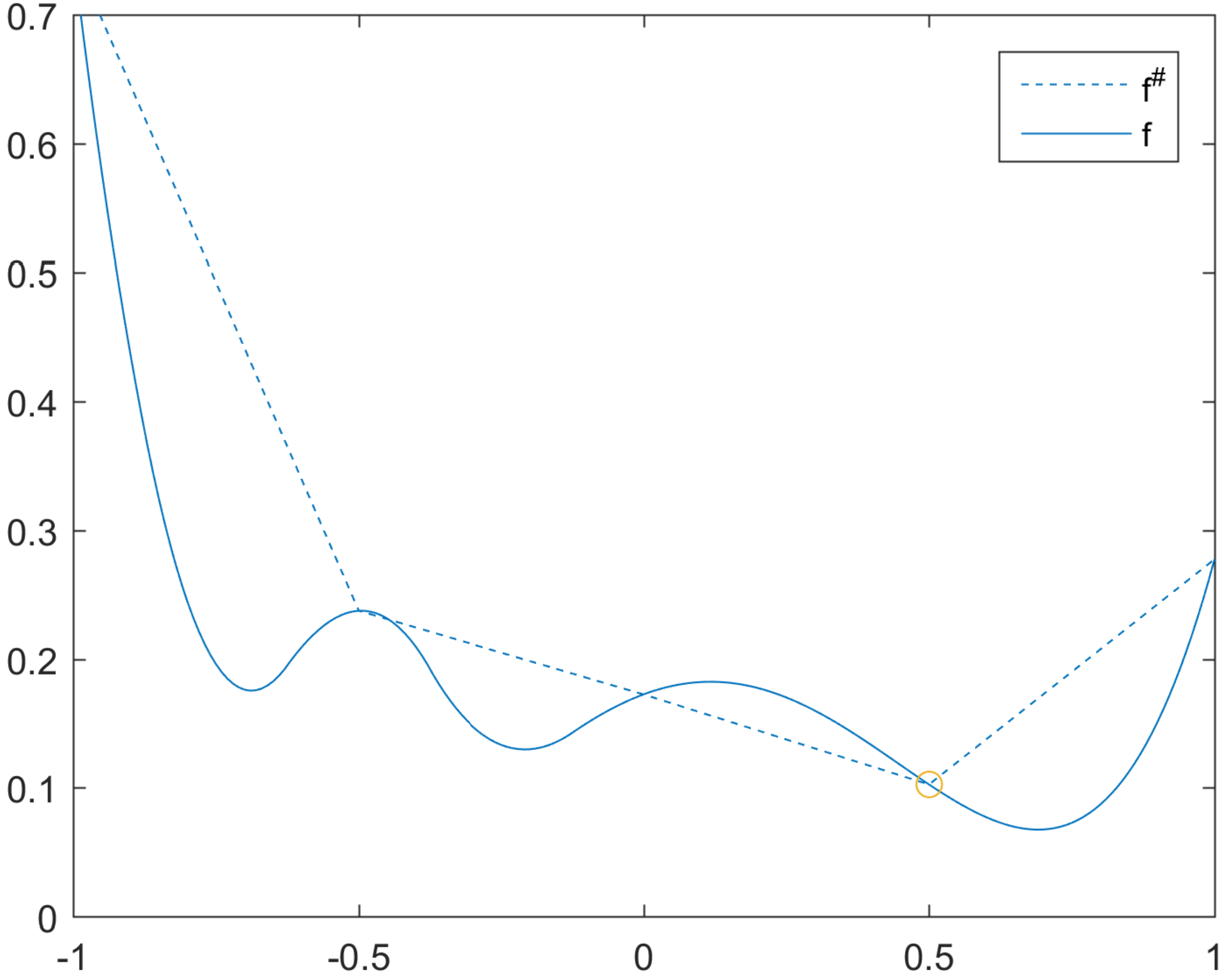
Lellmann, Lenzen, Schnörr, EMMCVPR '11, JMIV '13

Label bias



The solution tends strongly towards the chosen labels!

Lifting + linear relaxation



Precise relaxation

- Lifting + relaxation using the biconjugate:

$$\int_\Omega \rho(u(x))dx \rightsquigarrow \int_\Omega \boldsymbol{\rho}^{**}(\boldsymbol{u}(x))dx$$

Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16
related: Zach, Kohli, ECCV'12; Zach, AISTATS'13

Precise relaxation

- Lifting + relaxation using the biconjugate:

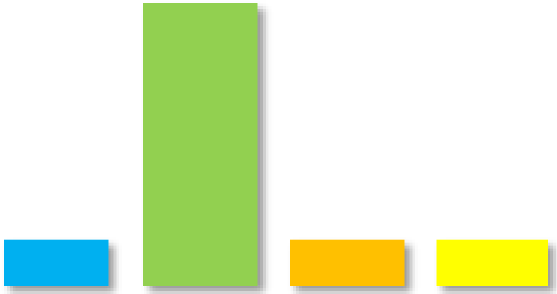
$$\int_\Omega \rho(u(x))dx \rightsquigarrow \int_\Omega \boldsymbol{\rho}^{**}(\boldsymbol{u}(x))dx$$

- Linear relaxation (*1-sparse* solutions):

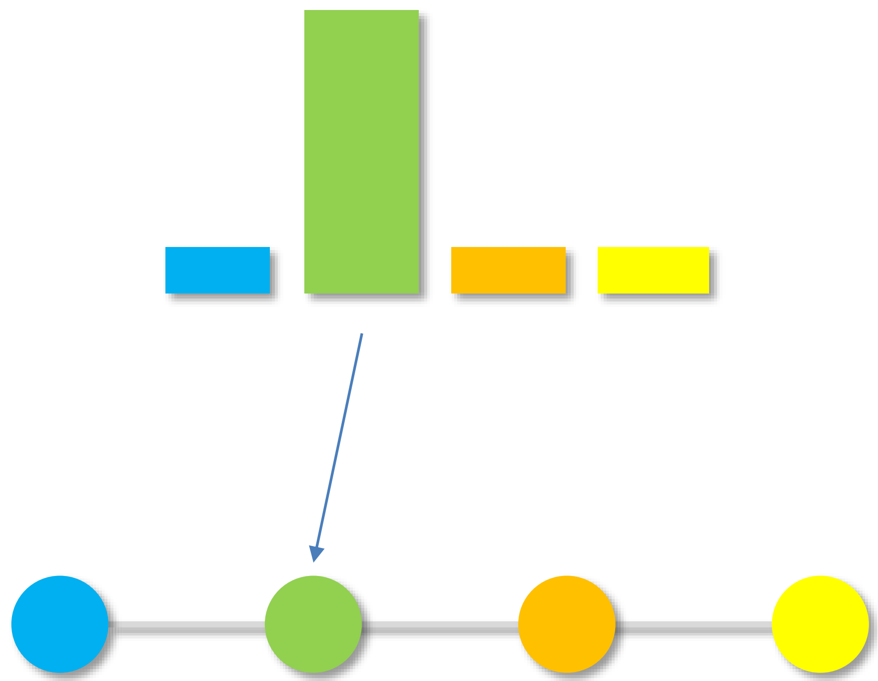
$$\boldsymbol{\rho}(z) = \begin{cases} \rho(t^i), & z = e^i,\ i \in \{1, \ldots, L\}, \\ +\infty, & \text{otherwise.} \end{cases}$$

Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16
related: Zach, Kohli, ECCV'12; Zach, AISTATS'13

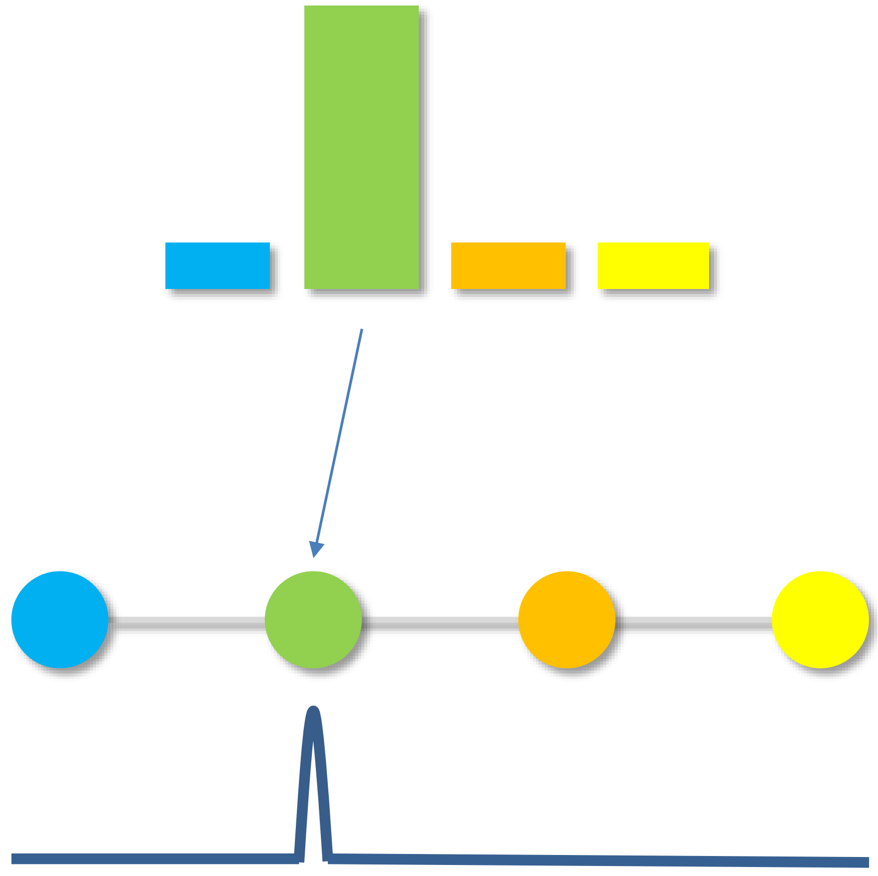
Non-binary values



Non-binary values



Non-binary values



Non-binary values

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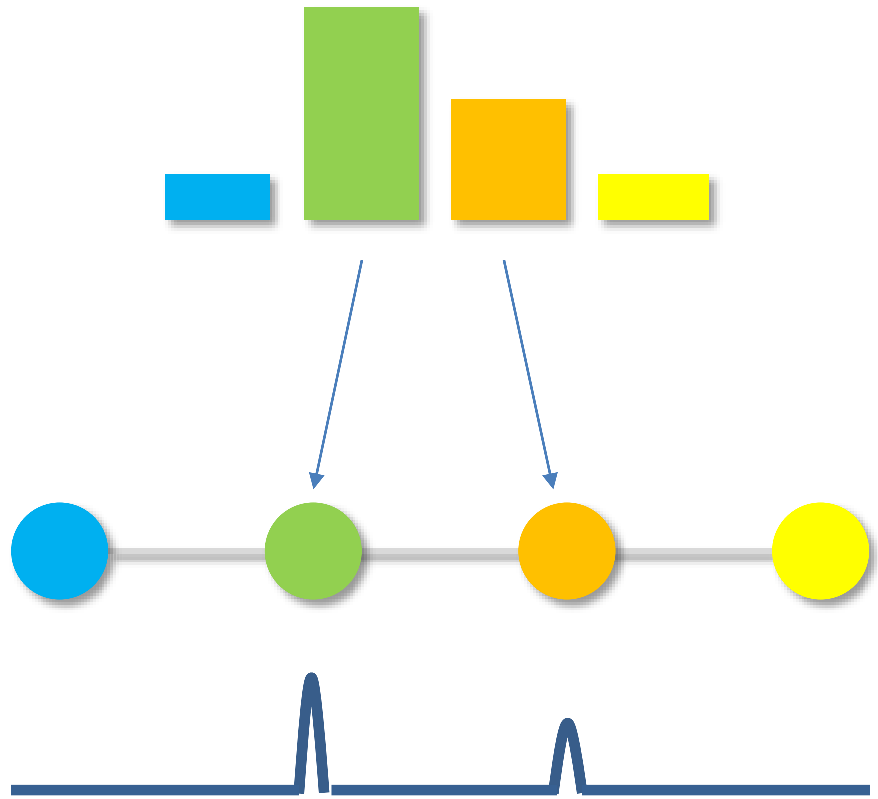

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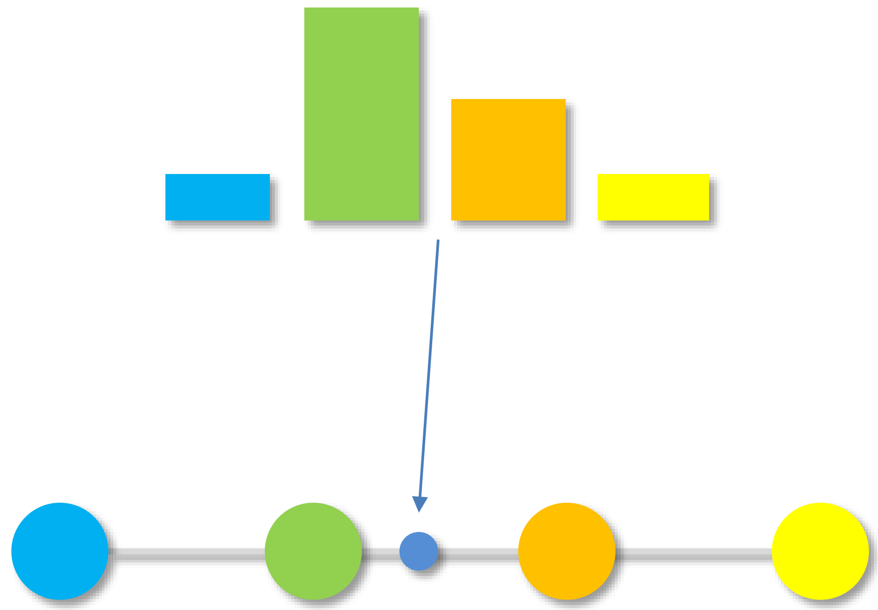
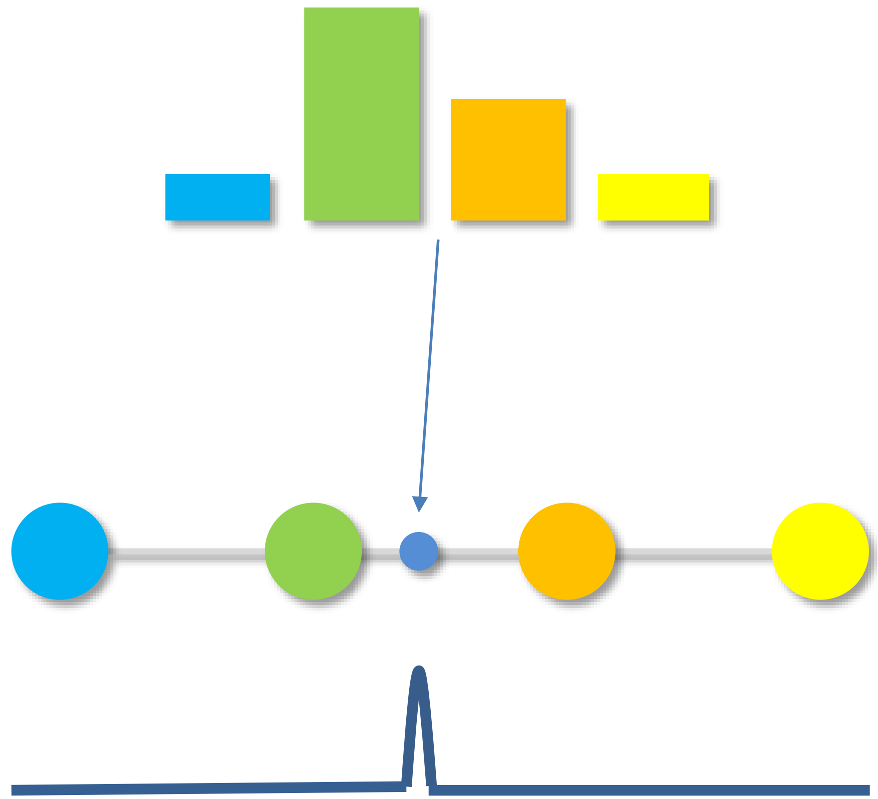


Non-binary values



Precise relaxation

- Lifting + relaxation using the biconjugate:

$$\int_\Omega \rho(u(x))dx \rightsquigarrow \int_\Omega \boldsymbol{\rho}^{**}(\boldsymbol{u}(x))dx$$

- New: Precise relaxation (*2-sparse*!)

$$\boldsymbol{\rho}(z) = \begin{cases} \rho((1-\alpha)t^i + \alpha t^{i+1}), & z = (1-\alpha)e^i + \alpha e^{i+1}, \\ +\infty, & \text{otherwise.} \end{cases}$$

- Linear programs: solution lies on vertex
- Here: solution between two vertices

Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16
related: Zach, Kohli, ECCV'12; Zach, AISTATS'13

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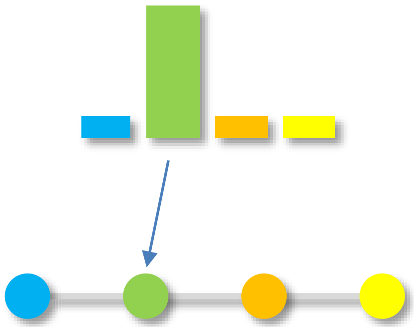

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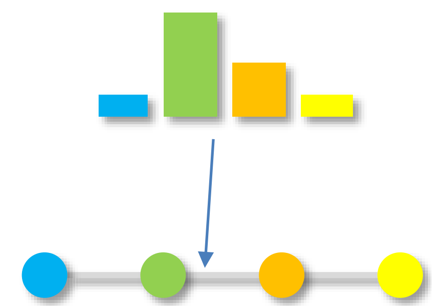

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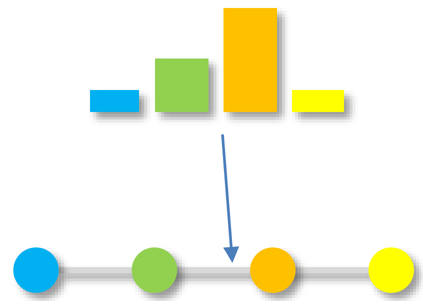

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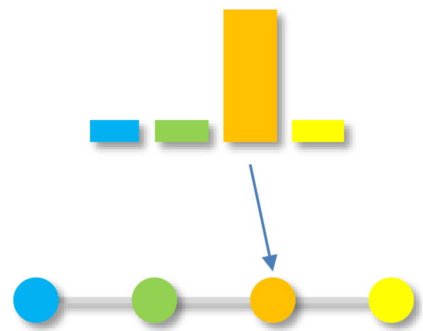

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Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16
related: Zach, Kohli, ECCV'12; Zach, AISTATS'13

Relaxing the regularizer – TV

Proposition 4. *The convex envelope of (15) is*

$$\Phi^{**}(\boldsymbol{g}) = \sup_{\boldsymbol{q} \in \mathcal{K}} \langle \boldsymbol{q}, \boldsymbol{g} \rangle, \tag{17}$$

where $\mathcal{K} \subset \mathbb{R}^{k \times d}$ *is given as:*

$$\begin{aligned} \mathcal{K} = \Big\{ \boldsymbol{q} \in \mathbb{R}^{k \times d} \;\Big|\; & \\ & \left| \boldsymbol{q}^\mathsf{T} (\mathbf{1}_i^\alpha - \mathbf{1}_j^\beta) \right|_2 \leq \left| \gamma_i^\alpha - \gamma_j^\beta \right|, \\ & \forall\, 1 \leq i \leq j \leq k,\ \forall \alpha, \beta \in [0,1] \Big\}. \end{aligned} \tag{18}$$

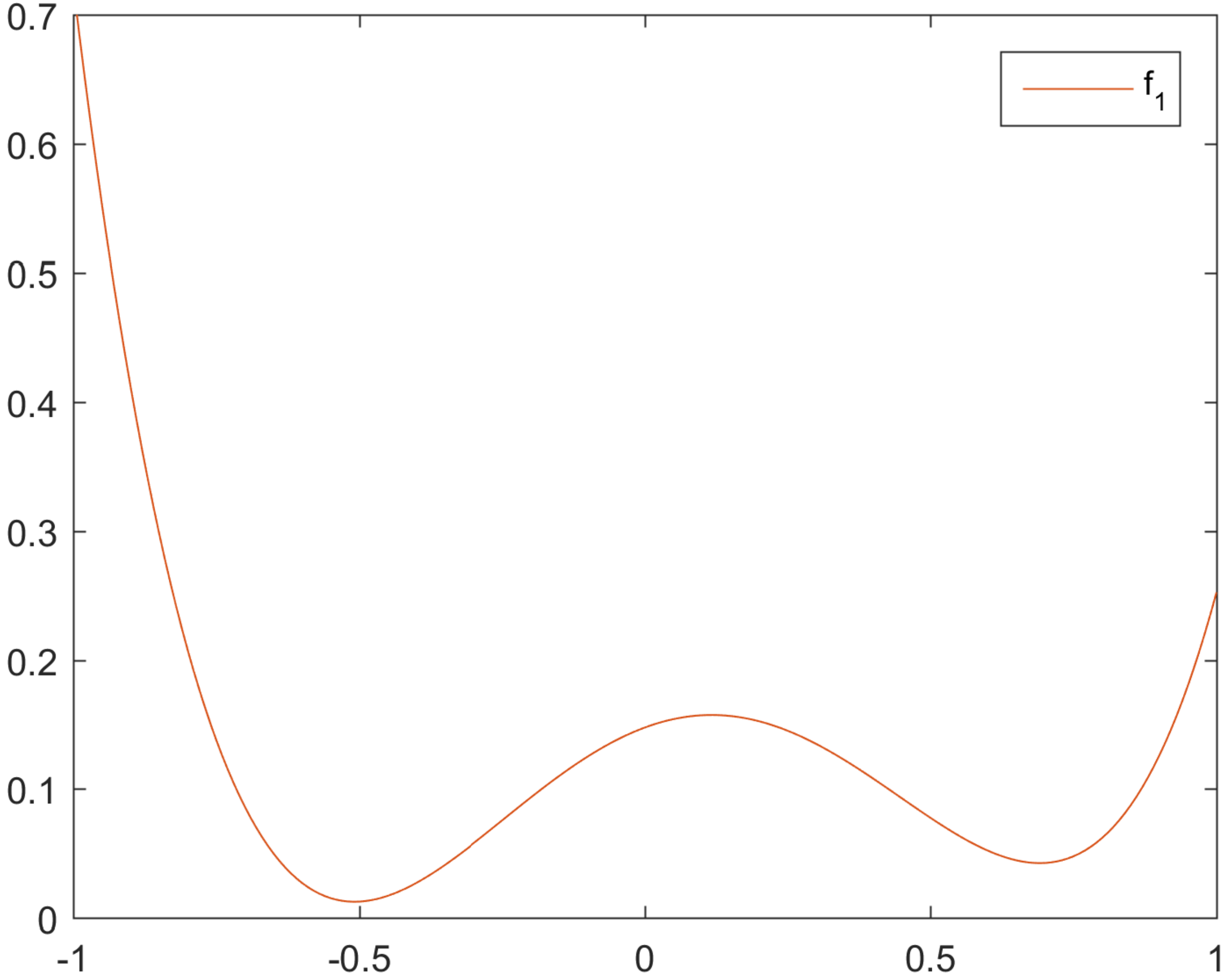
Proposition 5. *In case the labels are ordered, i.e.,* $\gamma_1 < \gamma_2 < \ldots < \gamma_L$, *then the constraint set* $\mathcal{K}$ *from Eq. (36) is equal to*

$$\mathcal{K} = \{ \boldsymbol{q} \in \mathbb{R}^{k \times d} \mid |\boldsymbol{q}_i|_2 \leq \gamma_{i+1} - \gamma_i,\ \forall i \}. \tag{19}$$

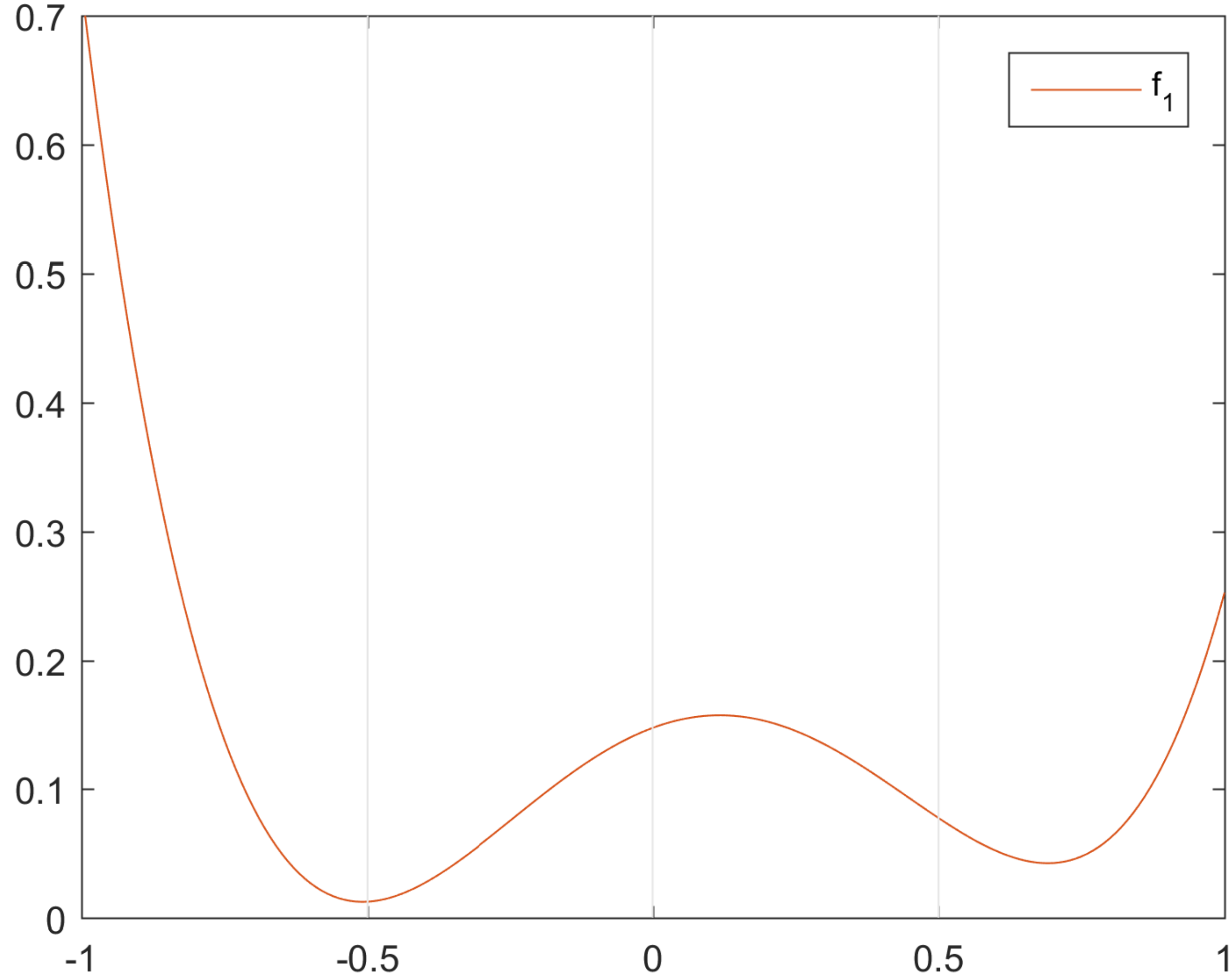
Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

New: lifting + precise relaxation

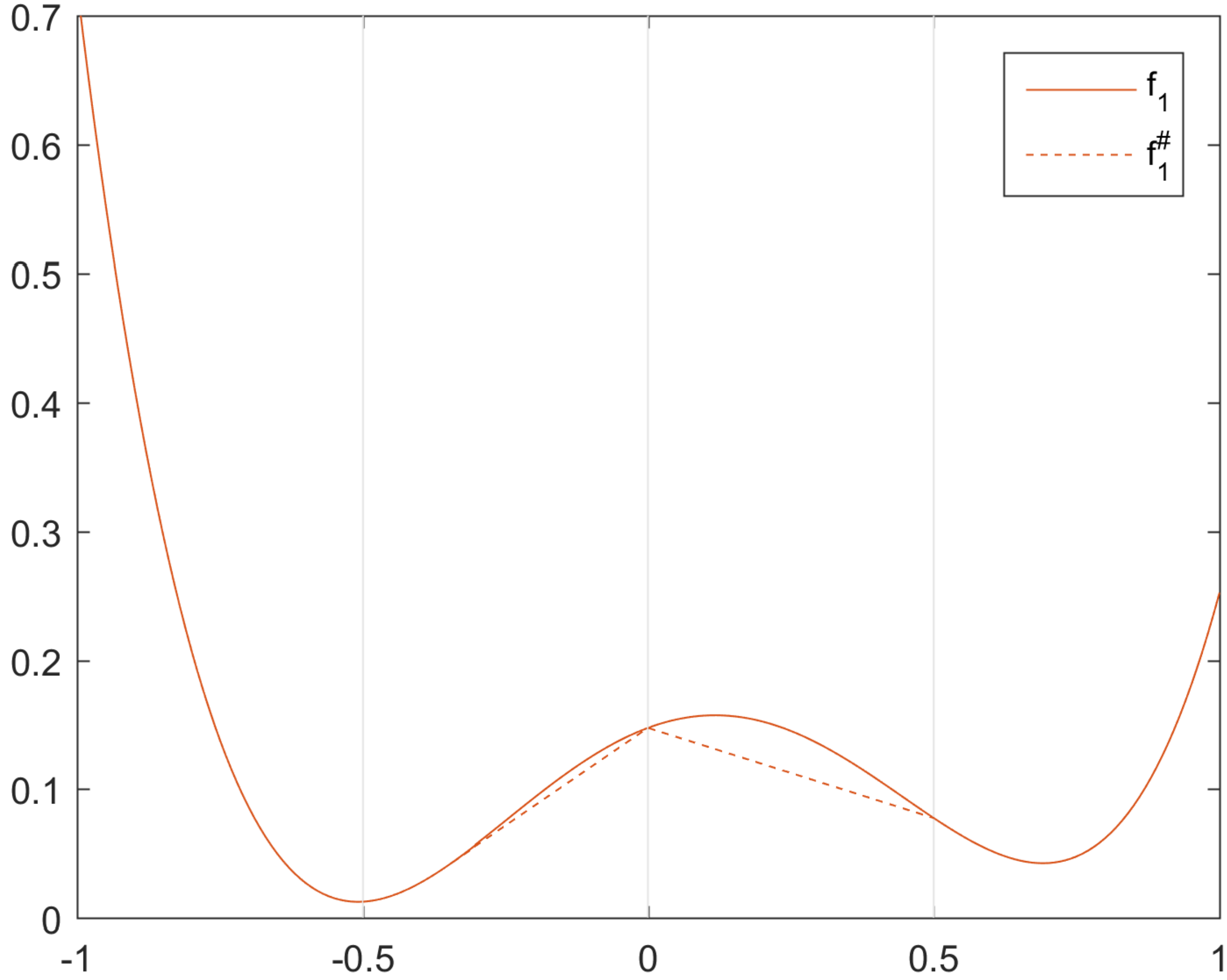
New: lifting + precise relaxation



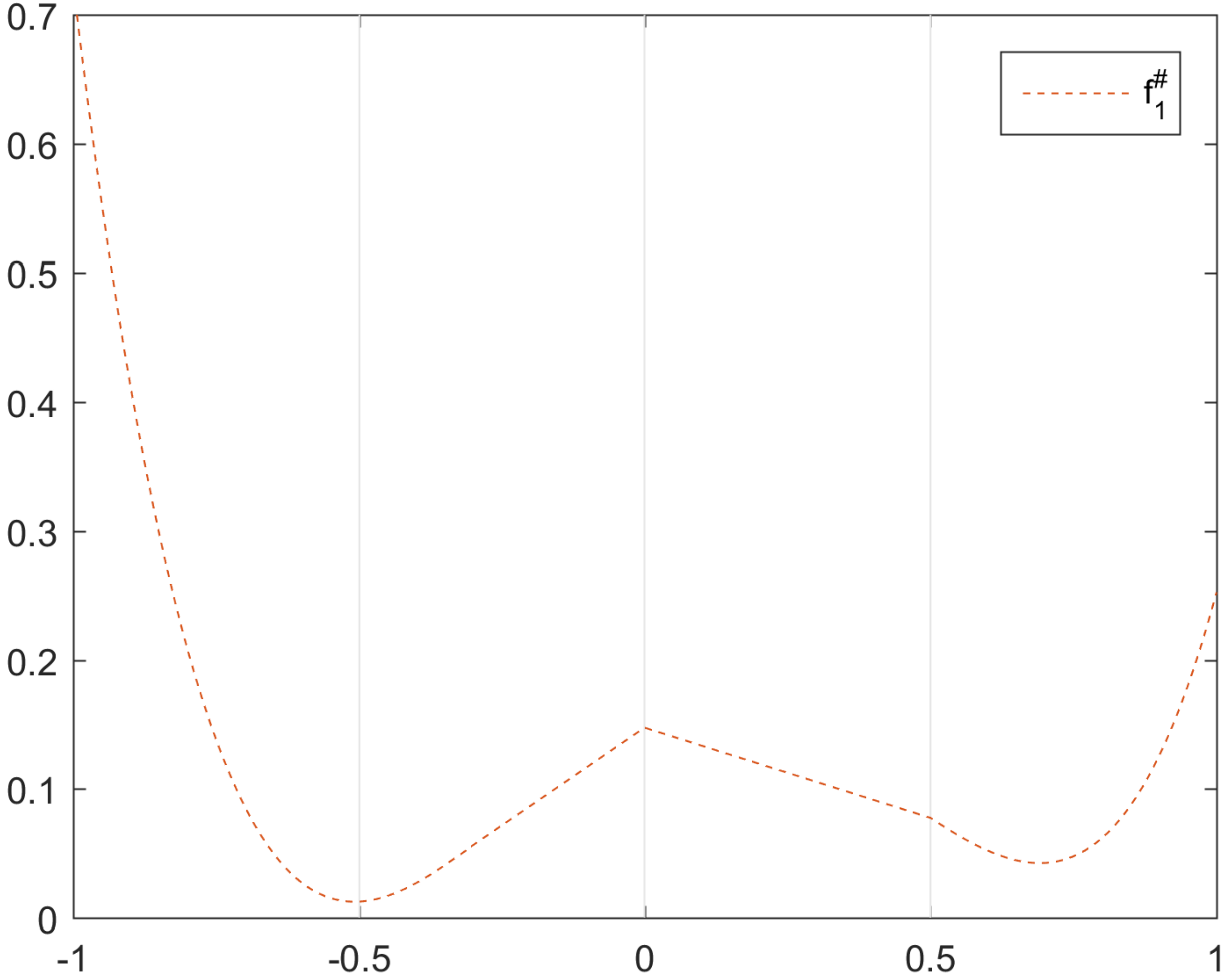
New: lifting + precise relaxation



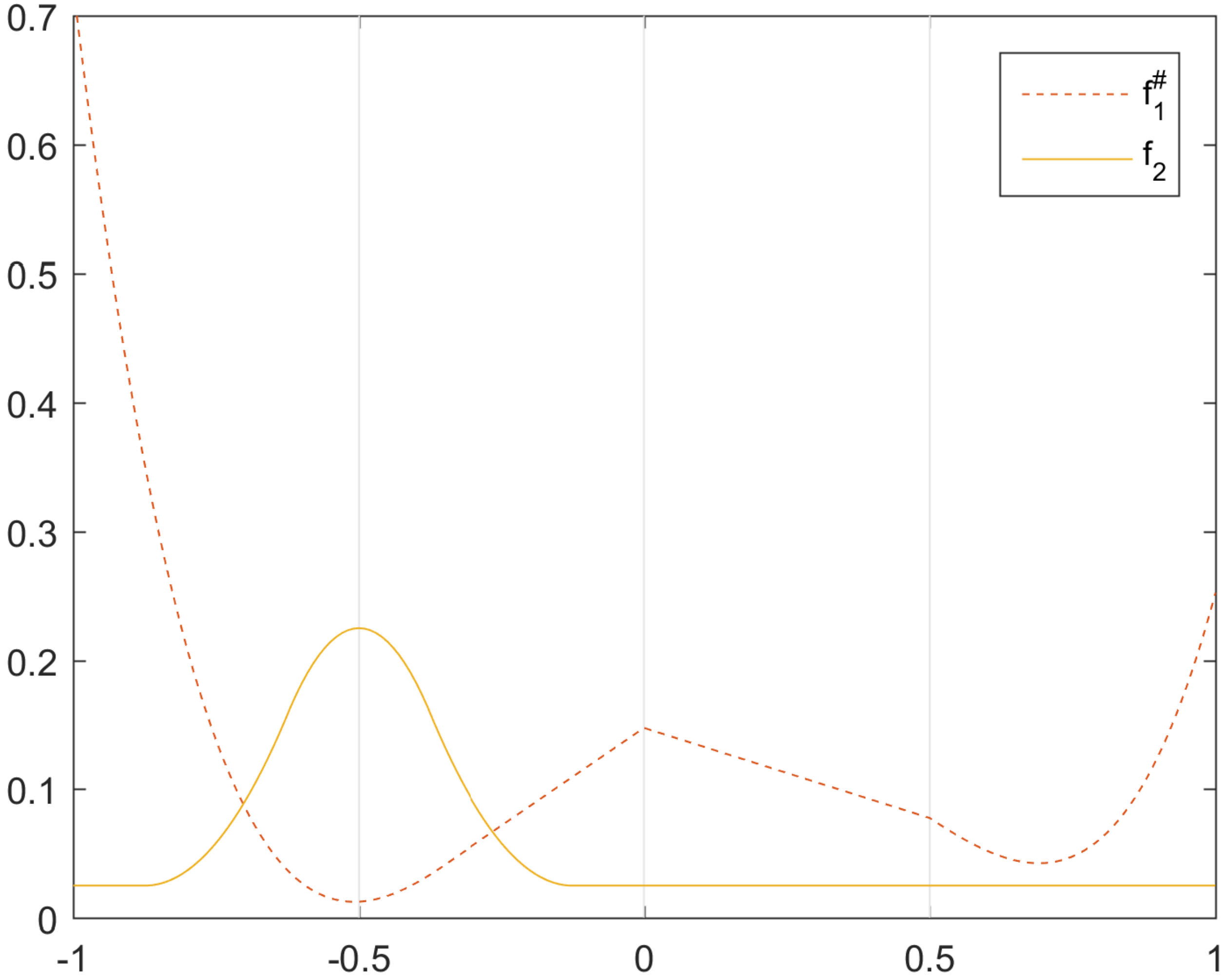
New: lifting + precise relaxation



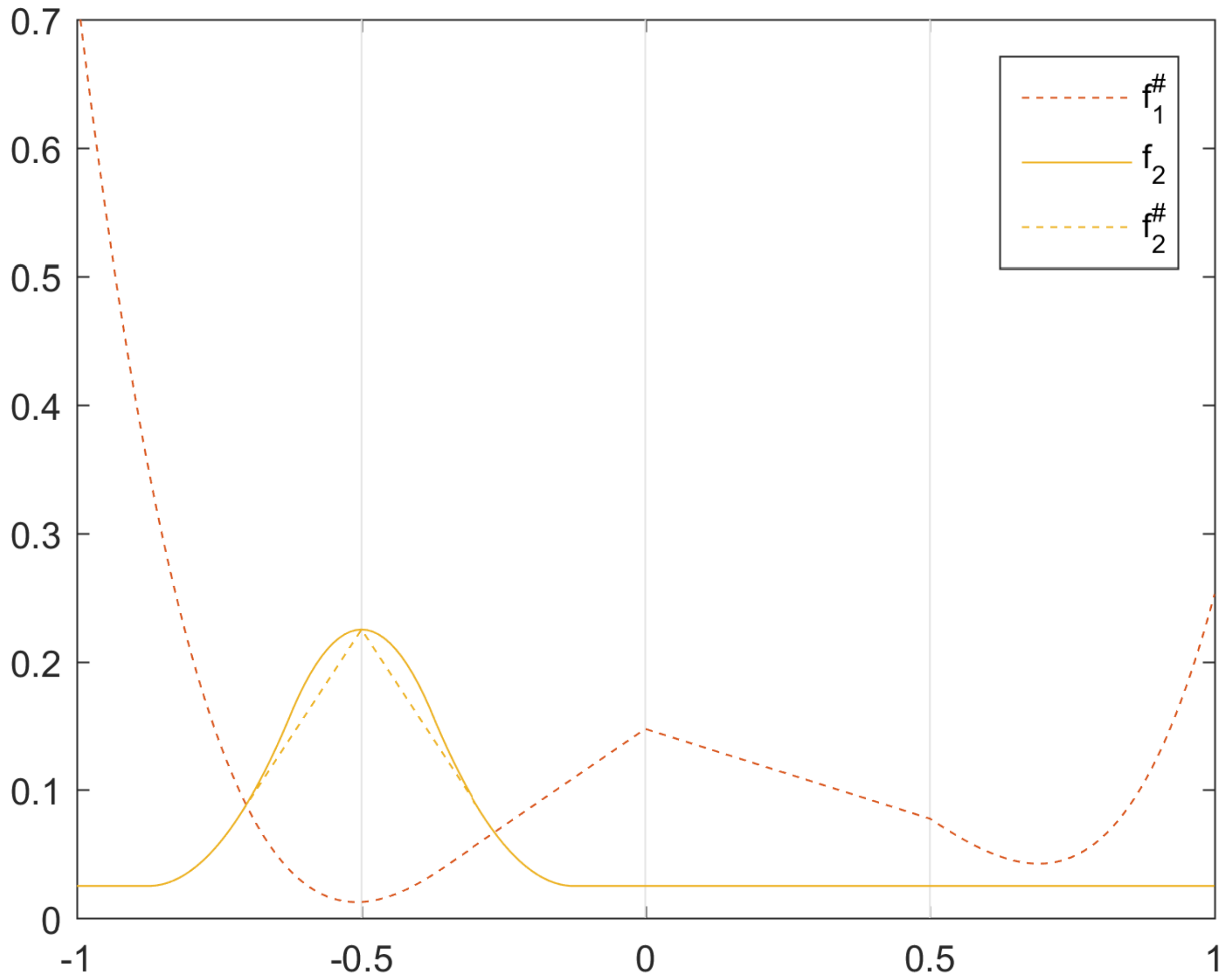
New: lifting + precise relaxation



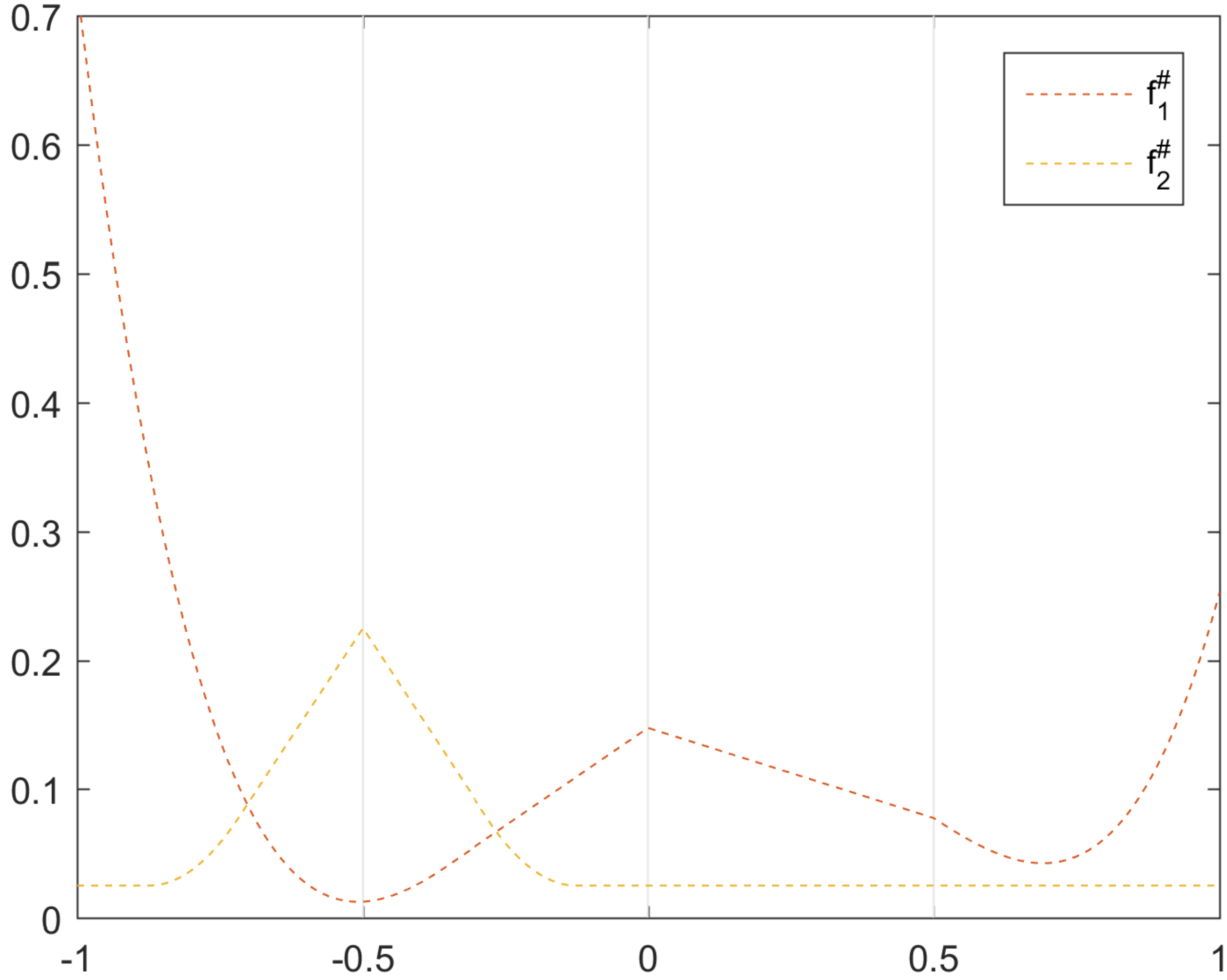
New: lifting + precise relaxation



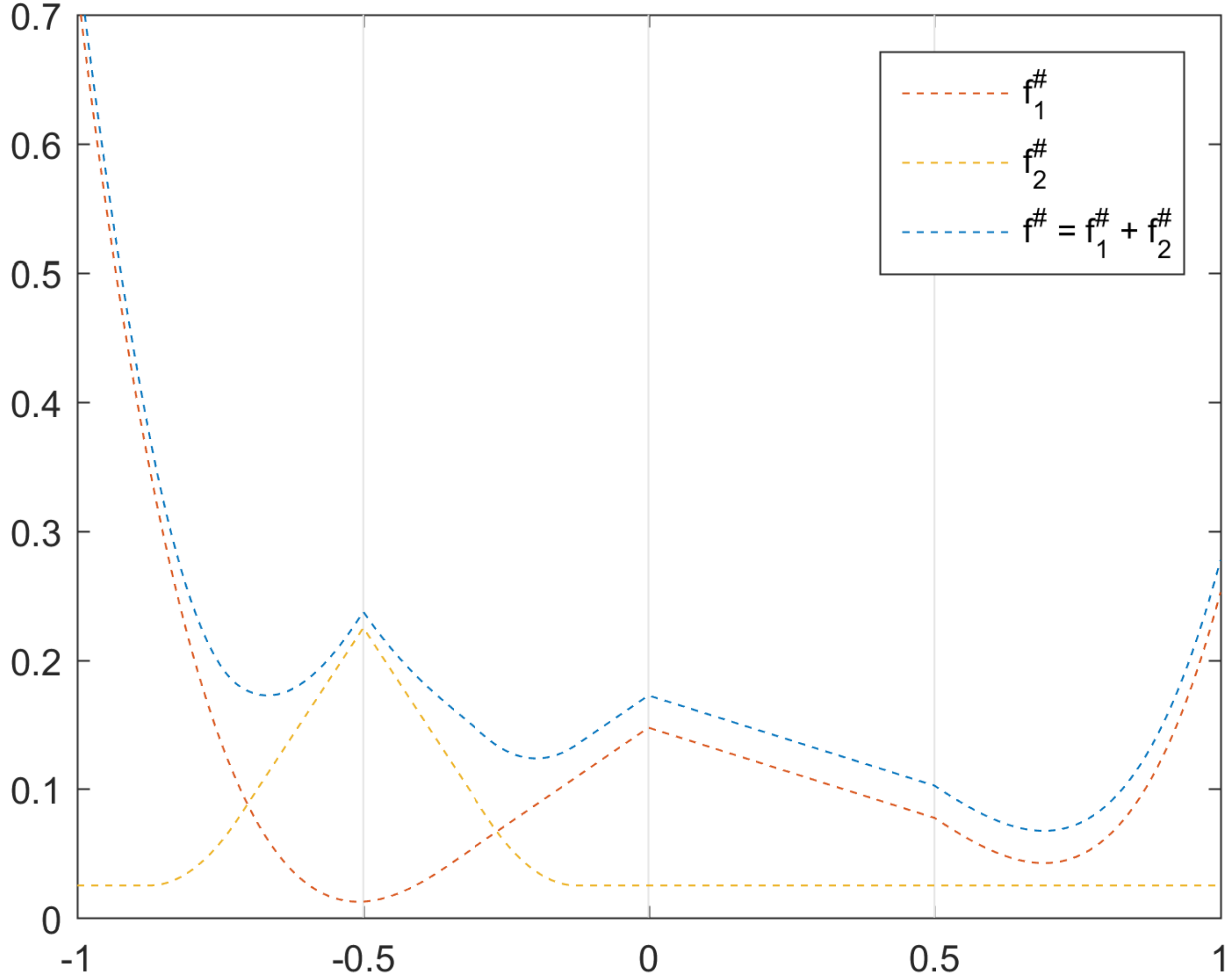
New: lifting + precise relaxation



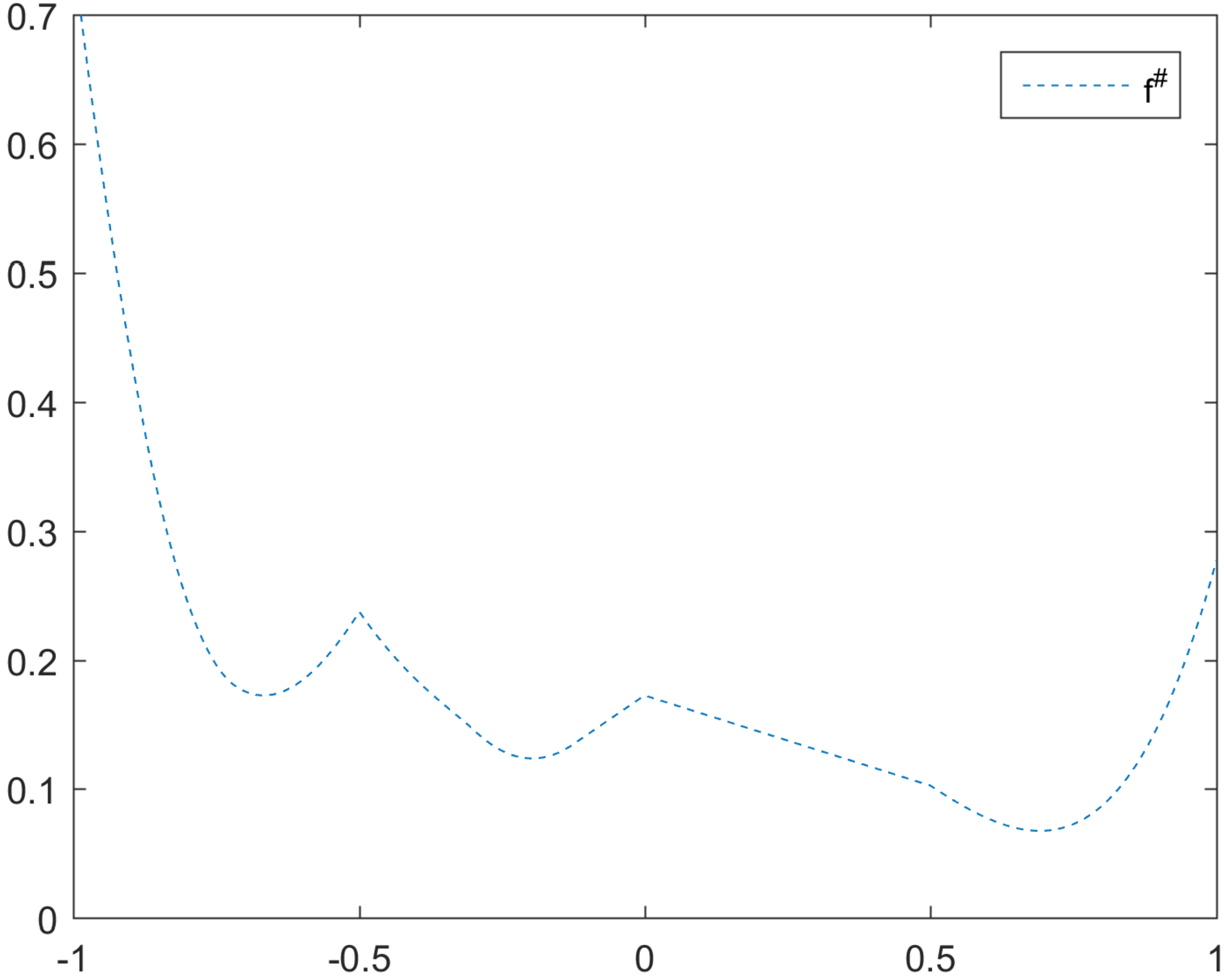
New: lifting + precise relaxation



New: lifting + precise relaxation



New: lifting + precise relaxation



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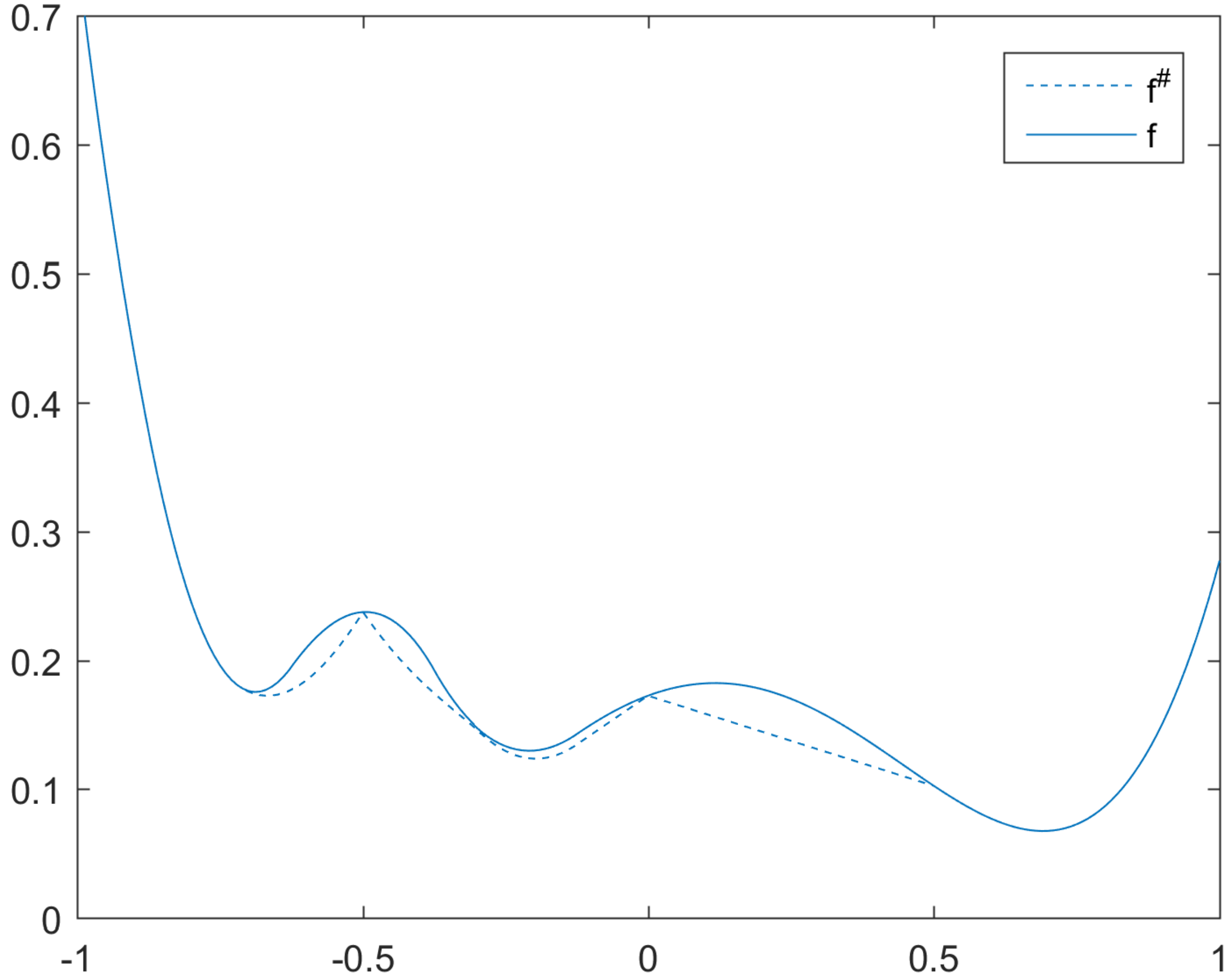
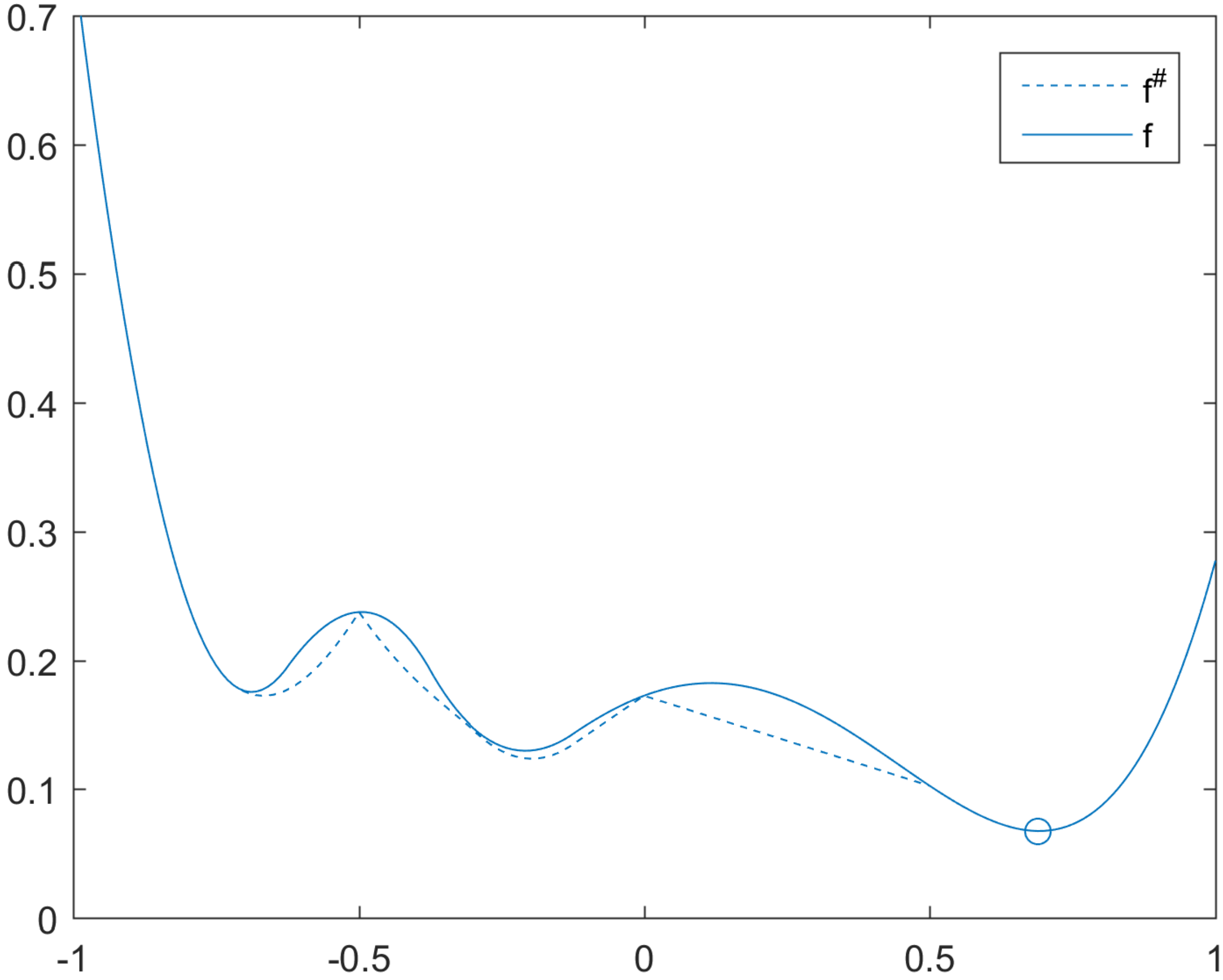


New: lifting + precise relaxation



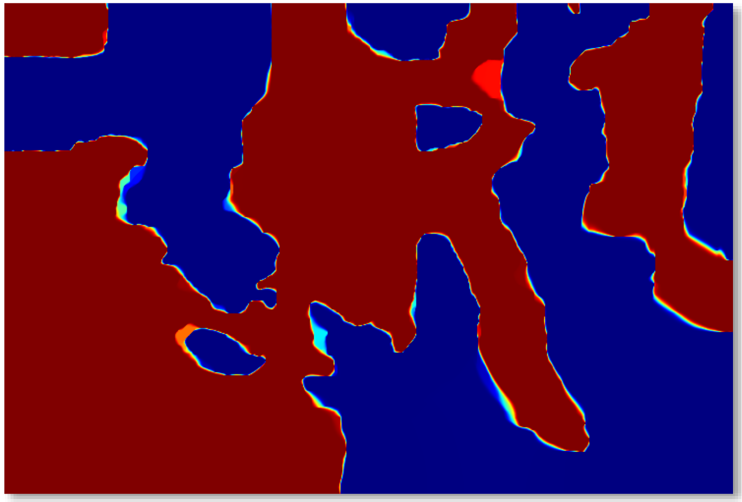
Results – disparity estimation

Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

linear lifting

$L = 2$

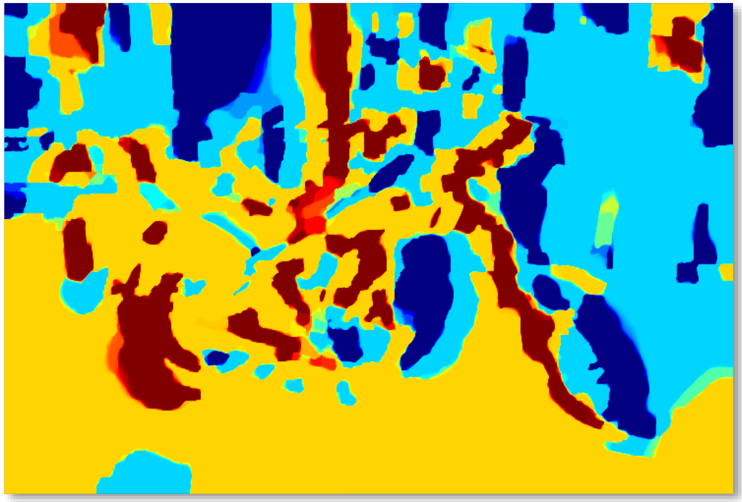


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

linear lifting

$L = 4$

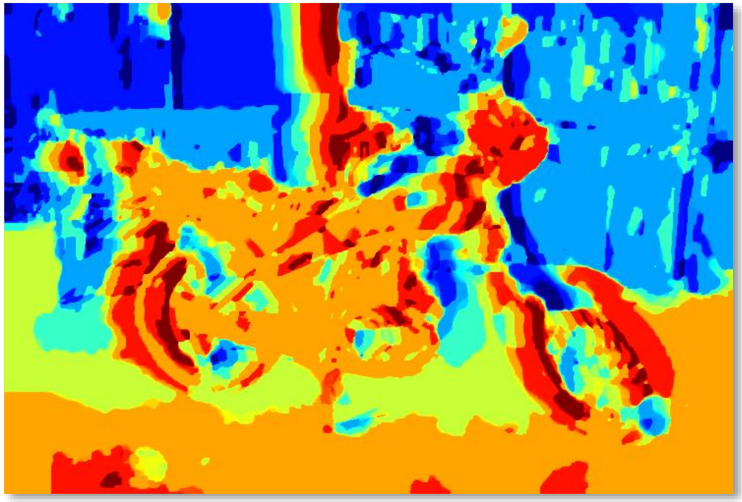


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

linear lifting

$L = 8$

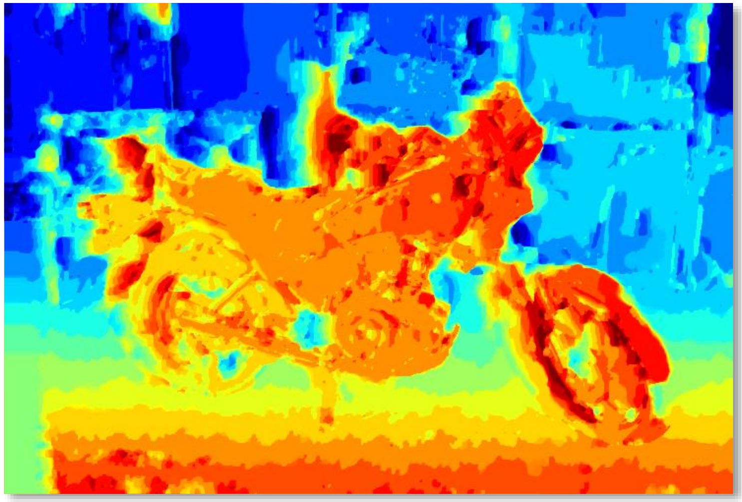


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

linear lifting

$L = 16$

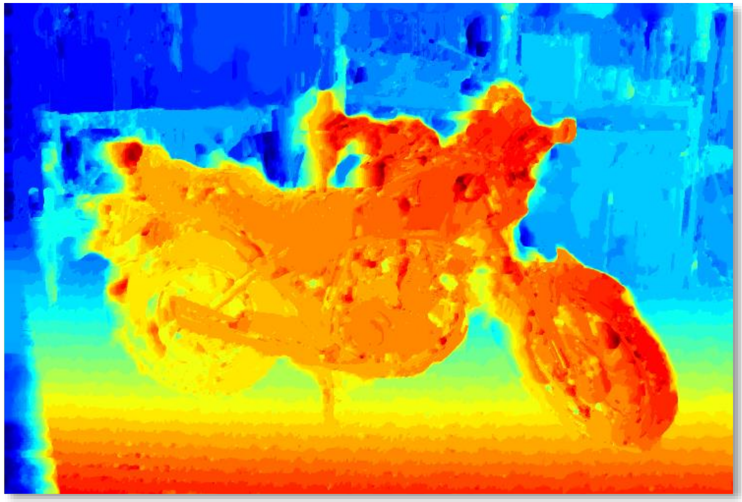


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

linear lifting

$$L = 32$$

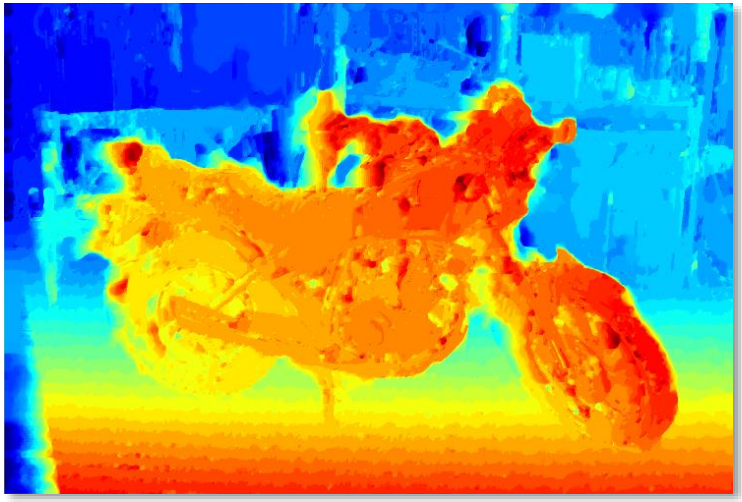


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

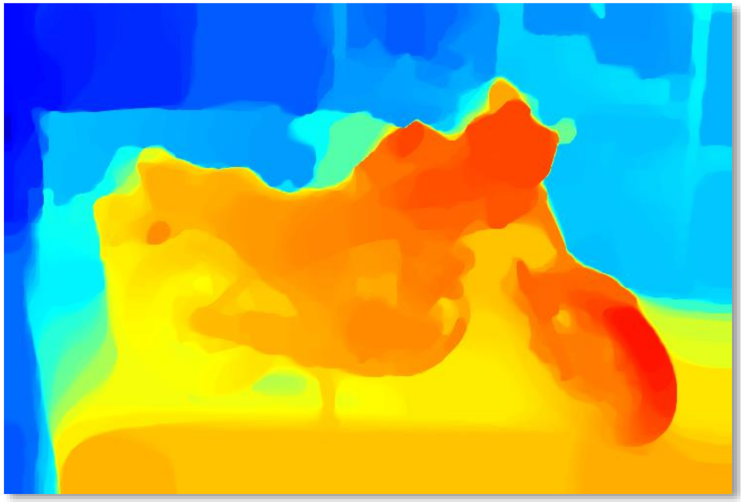
linear lifting

$L = 32$



precise lifting

$L = 2$



Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

linear lifting

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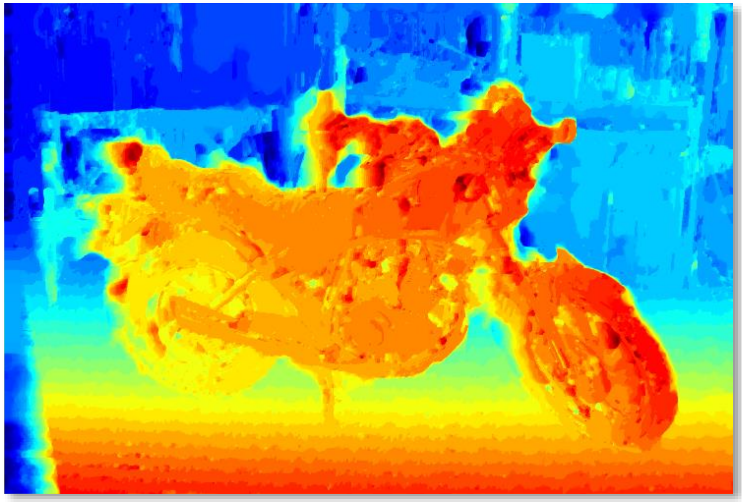
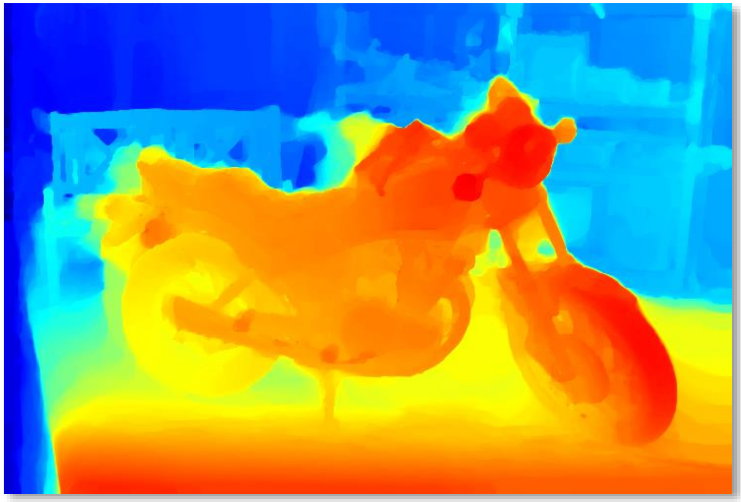


precise lifting

$L = 4$

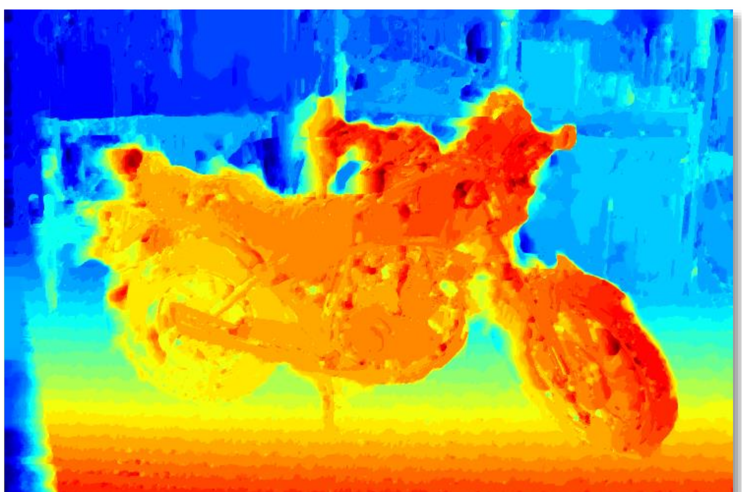


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

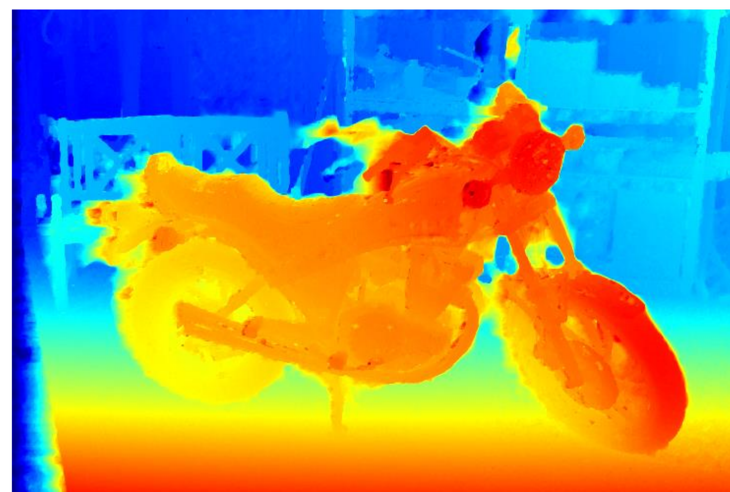
linear lifting

$L = 32$



precise lifting

$L = 8$

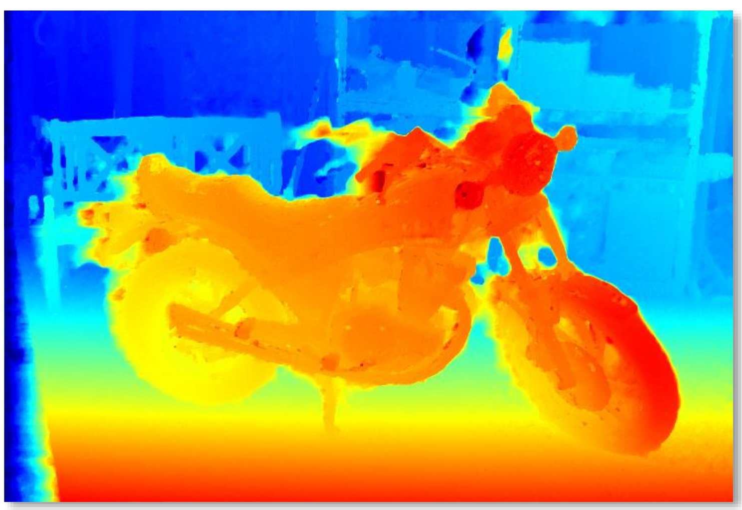


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR'16

Results – disparity estimation

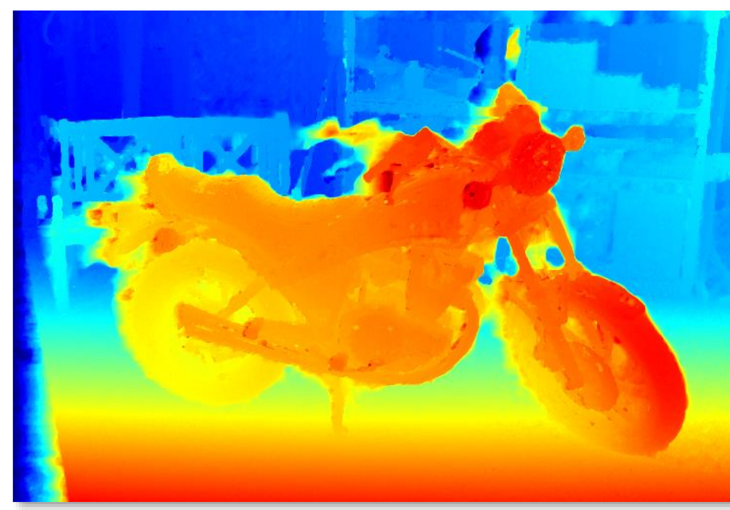
linear lifting

$L = 270$



precise lifting

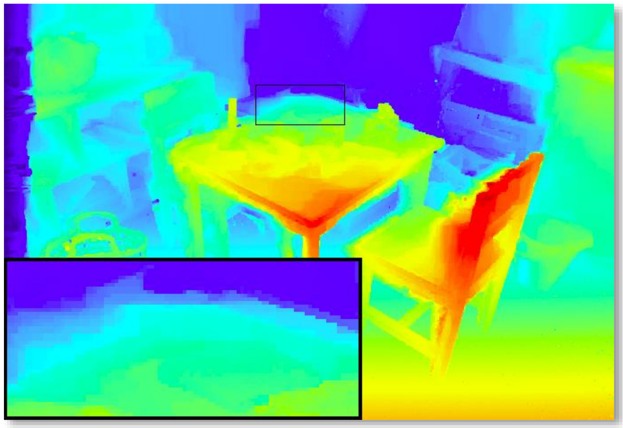
$L = 8$

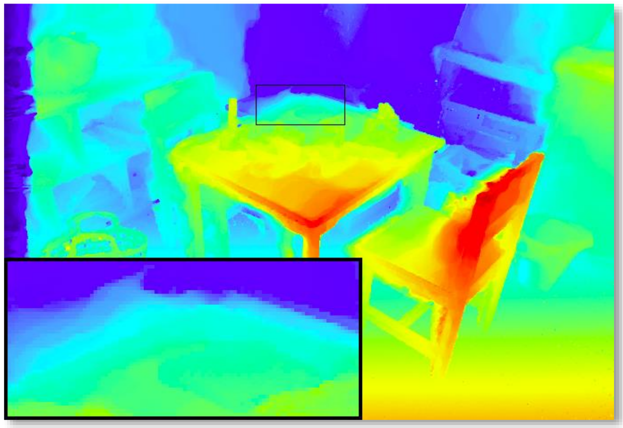


Möllenhoff, Laude, Möller, Lellmann, Cremers, CVPR’16

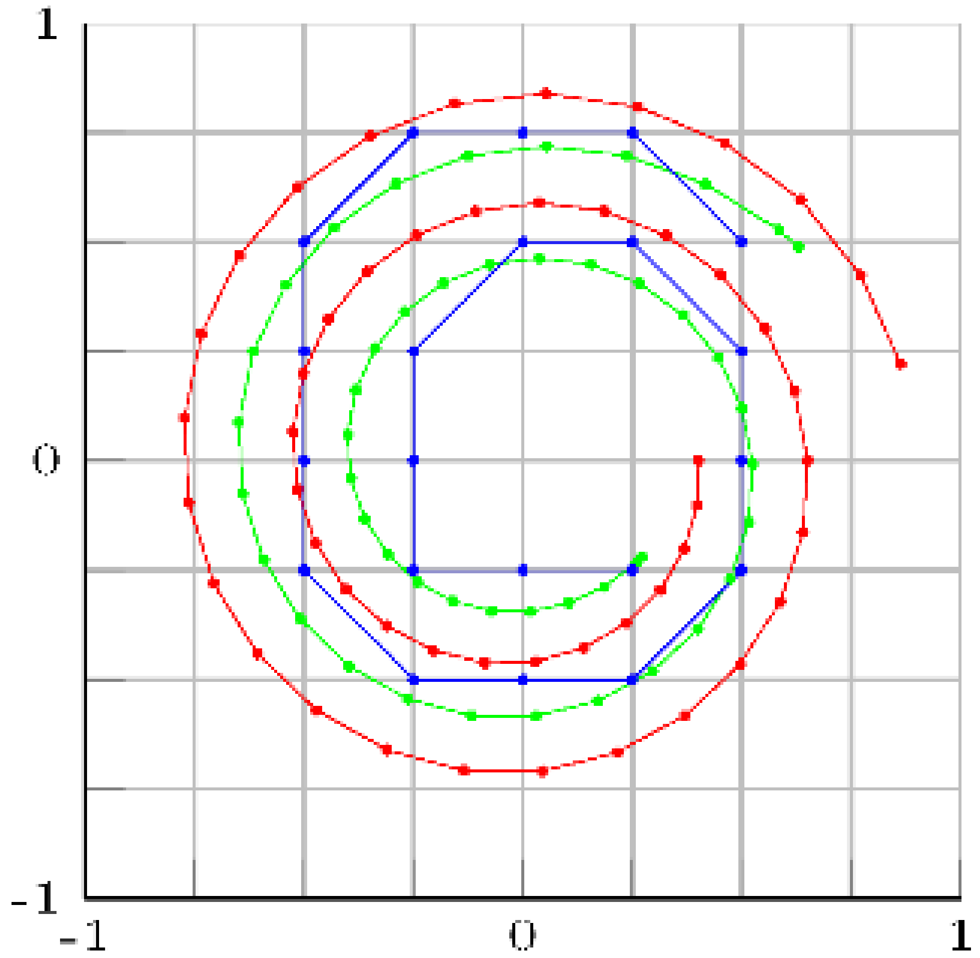
Related work

- Zach, Kohli’12; Zach’13; Fix, Agarwal’14
  - piecewise convex
  - different relaxation
  - MRF-based – not isotropic yet



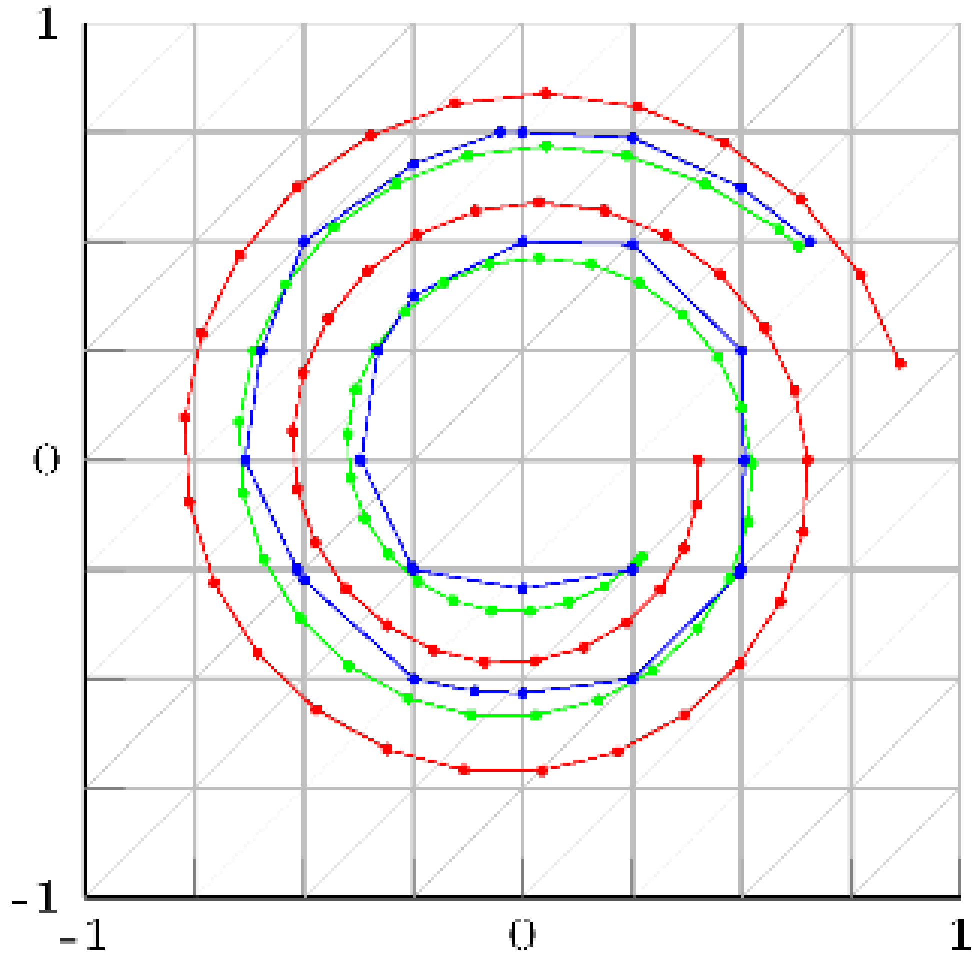


Multiple Dimensions



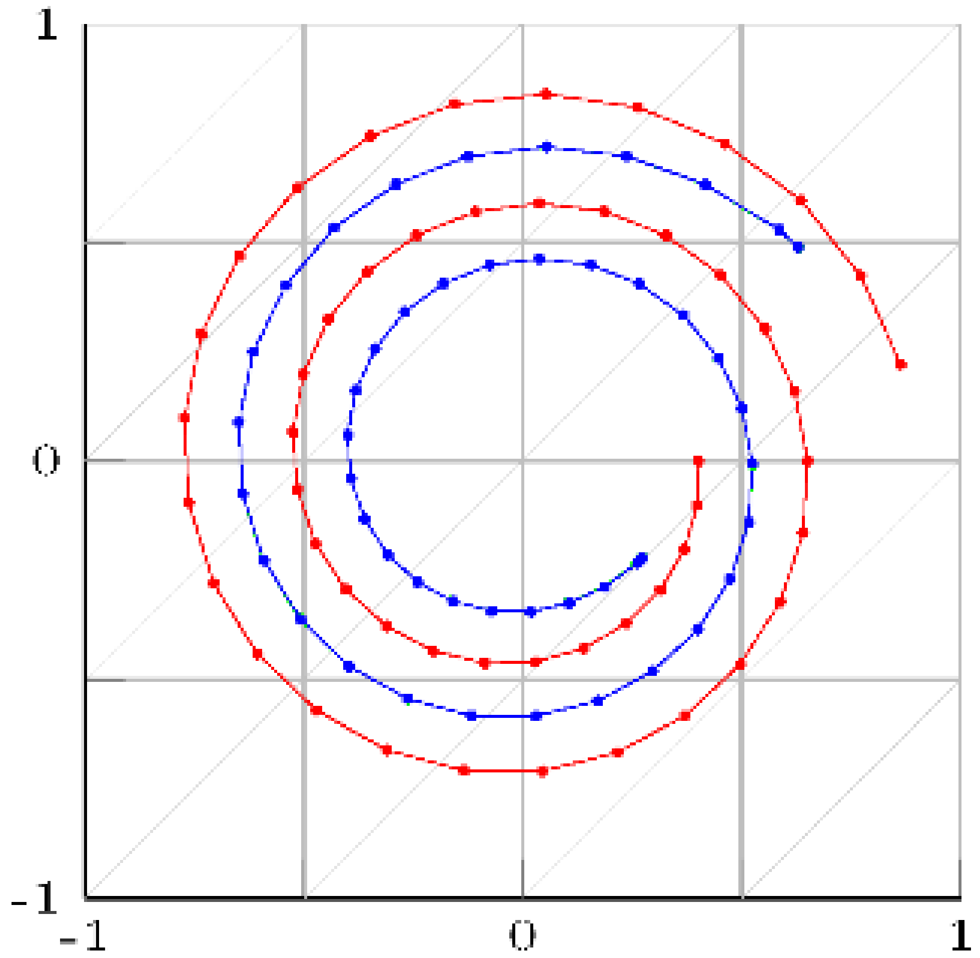
Laude, Möllenhoff, Möller, Lellmann, Cremers, ECCV'16

Multiple Dimensions



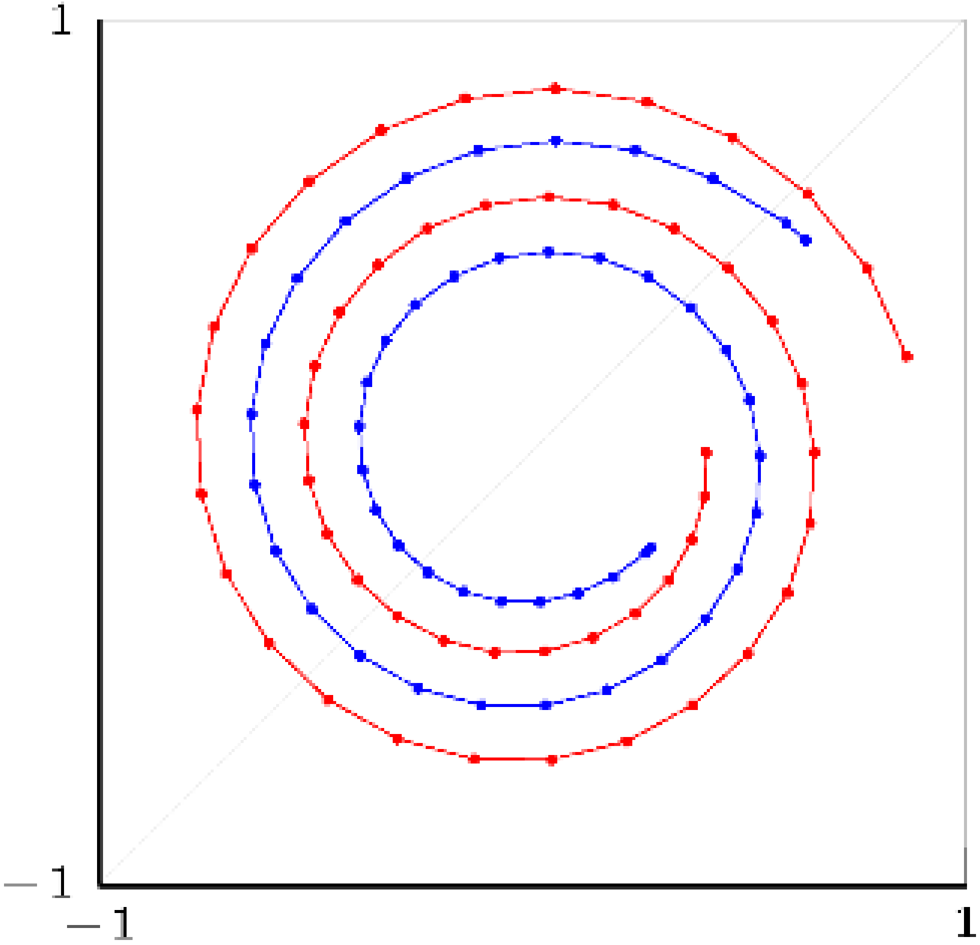
Laude, Möllenhoff, Möller, Lellmann, Cremers, ECCV’16

Multiple Dimensions



Laude, Möllenhoff, Möller, Lellmann, Cremers, ECCV'16

Multiple Dimensions



Laude, Möllenhoff, Möller, Lellmann, Cremers, ECCV'16

Color denoising

input

Möllenhoff, Laude, Möller, Lellmann, Cremers, ECCV'16

Color denoising

input

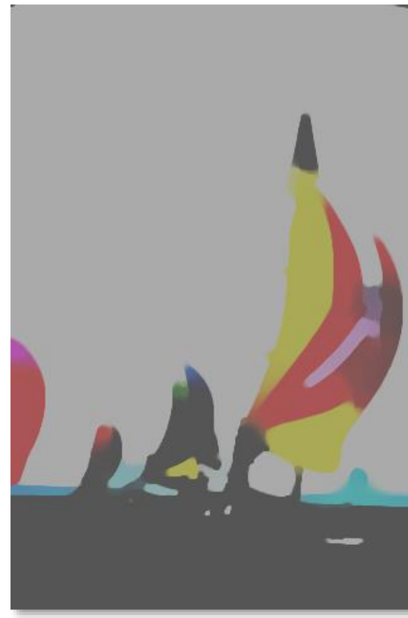
exact solution

Möllenhoff, Laude, Möller, Lellmann, Cremers, ECCV'16

Color denoising

input

exact solution



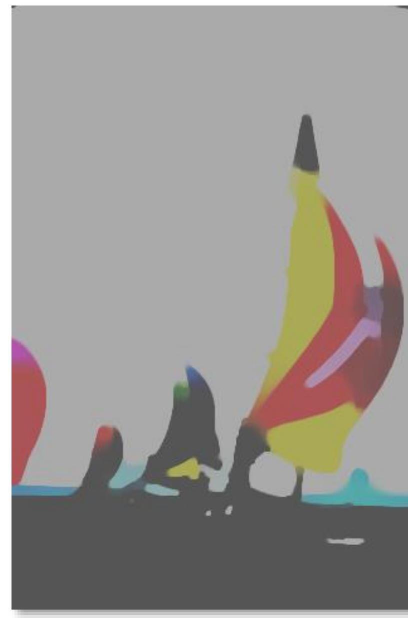
lifted linear
64 labels

Möllenhoff, Laude, Möller, Lellmann, Cremers, ECCV'16

Color denoising

input

exact solution



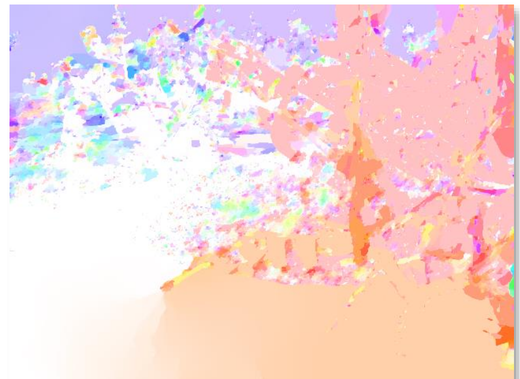
lifted linear
64 labels

lifted precise
8 labels

Möllenhoff, Laude, Möller, Lellmann, Cremers, ECCV'16

Large displacement optical flow

input sequence
(Middlebury)



LP-style relaxation (L. et al.'13)
7x7, 5.2GB, 33min, aep 2.65

ours, 2x2 labels
0.63GB, 17min, aep1.28

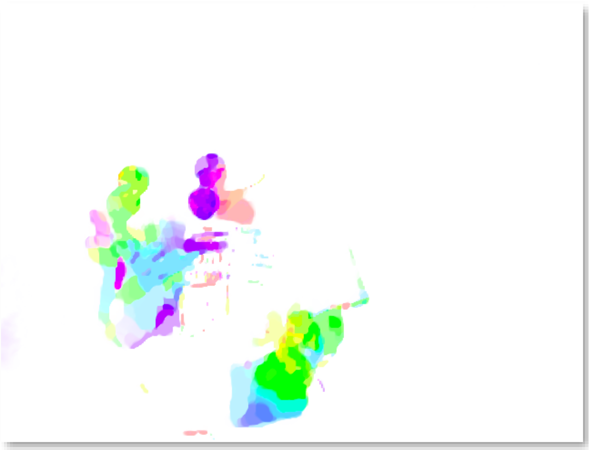
ground truth

product spaces (Goldluecke et al.'13),
28x28, 9.3GB, 60min, aep 1.39

ours, 6x6 labels
10.1GB, 56min, aep 0.9

Large displacement optical flow

input sequence



MRF-based, 81x81 labels

ours, 4 labels

References:

- T. Möllenhoff, E. Laude, M. Möller, J. Lellmann, D. Cremers: *Sublabel–Accurate Relaxation of Nonconvex Energies*. CVPR, 2016 (Best Paper Honorable Mention)
- E. Laude, T. Möllenhoff, M. Möller, J. Lellmann, D. Cremers: *Sublabel-Accurate Convex Relaxation of Vectorial Multilabel Energies*. arXiv Tech. Rep., 2016

Take-home

- Goal: global minimizer of nonconvex energies
- Lift into larger space
- Relax *piecewise convex*
- Much smaller problems, often 2-4 labels enough

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