Sublabel-Accurate Relaxation of Nonconvex Energies

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MATLAB code

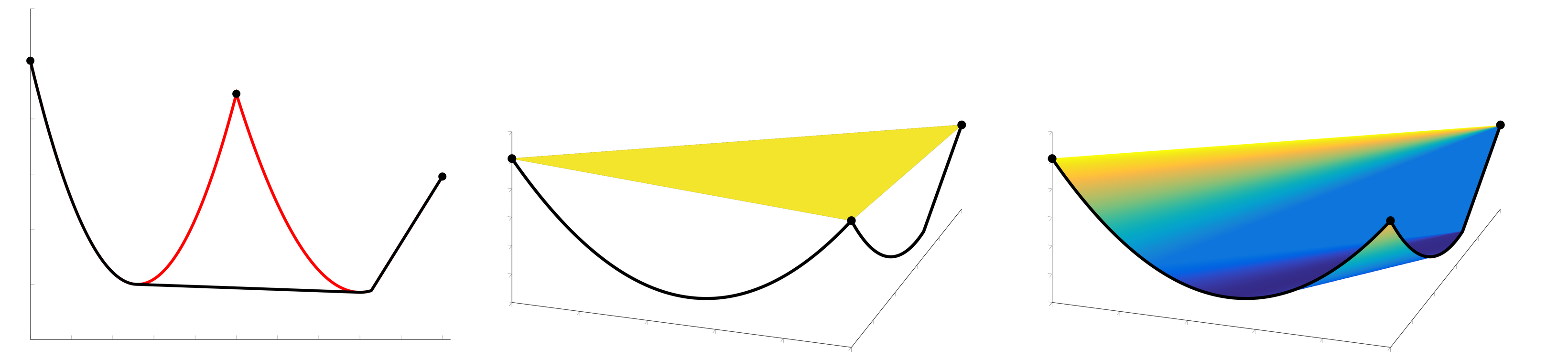
Problem Statement

- We consider **nonconvex energy minimization** problems

$$\min_{u:\Omega\to\Gamma} \int_\Omega \rho(x,u(x))\,\mathrm{d}x + \mathrm{TV}(u)$$

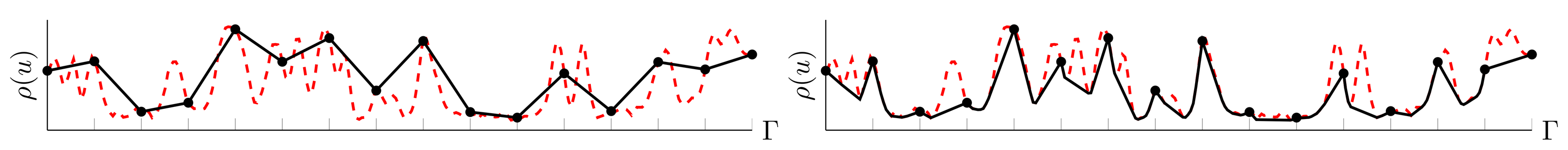
- **Main difficulties:**
  - Nonconvex data term $\rho : \Omega \times \Gamma \to \mathbb{R}$
  - Continuous range $\Gamma = [\gamma_{\min}, \gamma_{\max}]$
- **Contributions:**
  - Savings in memory and runtime over previous spatially continuous approaches such as [1]
  - We compute the provably tightest local convex relaxation
  - First spatially continuous sublabel accurate multi-labeling, no grid bias compared to [3]

Convexification Framework



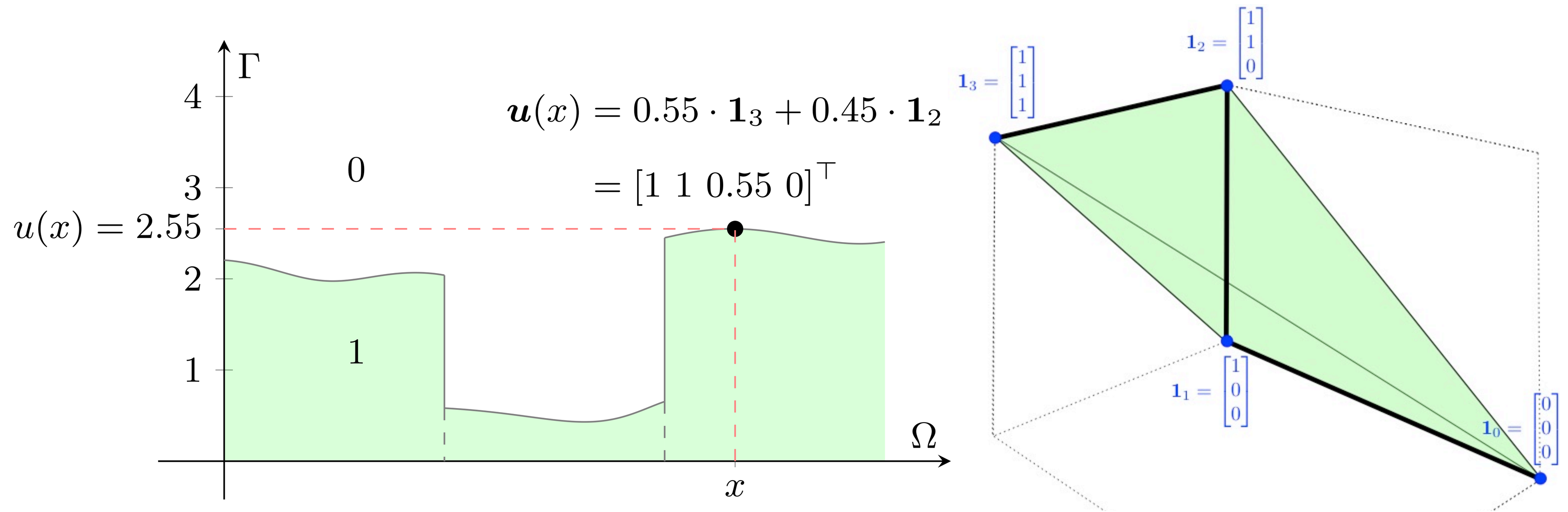
Without Lifting | Standard Lifting [1] | Proposed approach

- Direct convexification leads to loss of information
- **Main idea:** compute convex extension in higher dimensional space
- Standard approaches such as [1] specify cost at the labels
- Our framework can handle convex costs also **between** the labels



Standard Lifting [1] | Proposed approach

- Consider discretization of graph functions in $\Omega \times \Gamma$:



$$\boldsymbol{\rho}(x, \mathbf{u}(x)) = \begin{cases} \rho(x, \gamma_i^\alpha) & \text{if } \mathbf{u}(x) = \mathbf{1}_i^\alpha, \\ +\infty & \text{otherwise.} \end{cases} \quad \boldsymbol{\rho}^{**}$$

Lifted Total Variation

- **Proposition:** the total variation for the lifted representation is

$$\mathbf{TV}(\boldsymbol{u}) = \sup_{\boldsymbol{p}\in K} \langle \boldsymbol{u}, \mathrm{Div}\, \boldsymbol{p} \rangle$$

$$K = \{\boldsymbol{p} : \Omega \to \mathbb{R}^{k\times d} \mid \|\boldsymbol{p}_i(x)\| \leq \gamma_{i+1} - \gamma_i, \forall i, \forall x\}$$

- This is the same as for the standard approach [1]

Primal-Dual Optimization

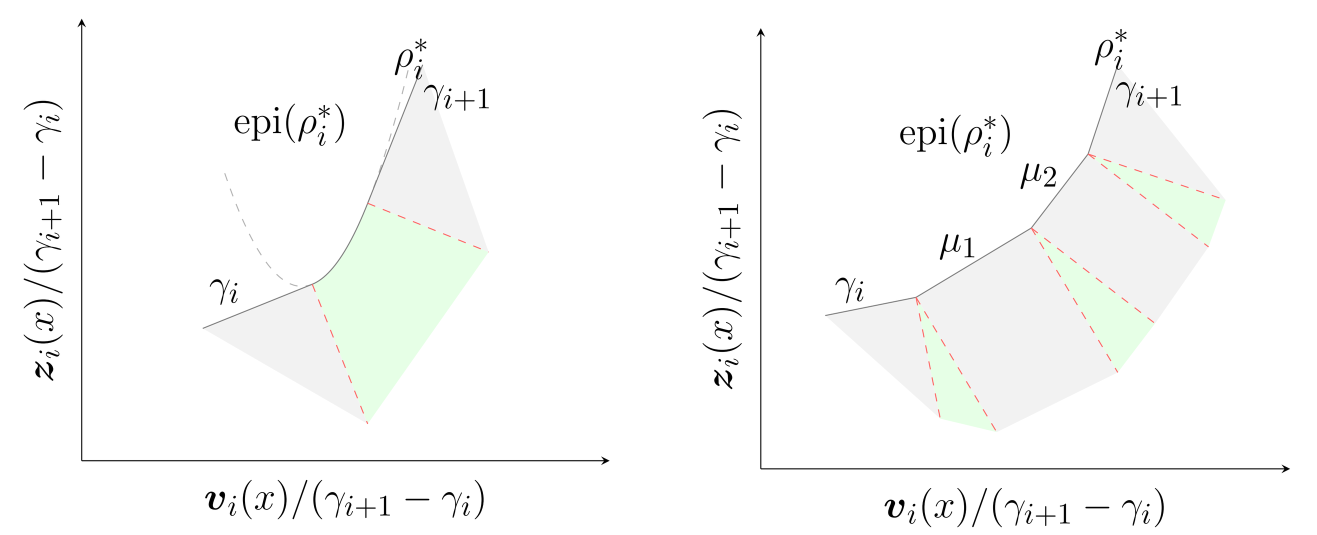
- Optimization problem is a convex-concave saddle-point problem

$$\min_{x\in H} \max_{y \in C} \langle Ax, y\rangle + \langle g, x\rangle - \langle h, x\rangle$$

- Can be solved using an efficient primal-dual algorithm [2]

$$x^{k+1} = \mathrm{proj}_H \left(x^k - \tau \left(A^\top y^k + g\right)\right)$$
$$y^{k+1} = \mathrm{proj}_C \left(y^k + \sigma \left(A(2x^{k+1} - x^k) - h\right)\right)$$

- **Central computation step:** orthogonal projection onto the **epigraph** of the Legendre-Fenchel conjugate of the convex pieces



- We have developed a general framework for optimization problems with such **pro**ximal **st**ructure
- Contains efficient and generic CUDA implementation of the primal-dual algorithm [2] and ADMM
- Easy to use MATLAB interface

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Proof of Concept

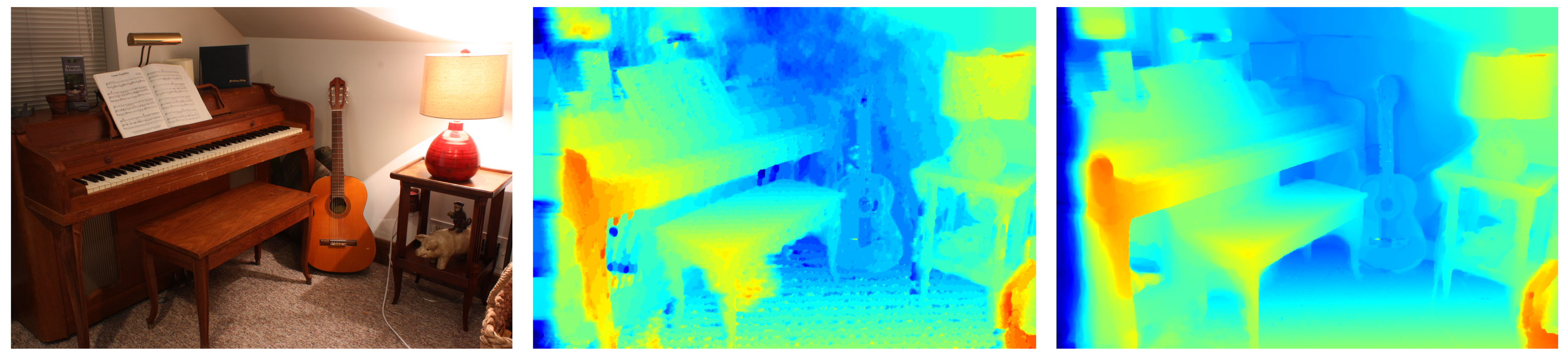
Convex ROF model: $\rho(x, u(x)) = (u(x) - f(x))^2$

Direct, t=0.6 s, 11.8 MB | Proposed, 2 labels, t=1 s, 27 MB | baseline [1], 8 labels, 226 MB

Exact solutions and small overhead compared to direct optimization!

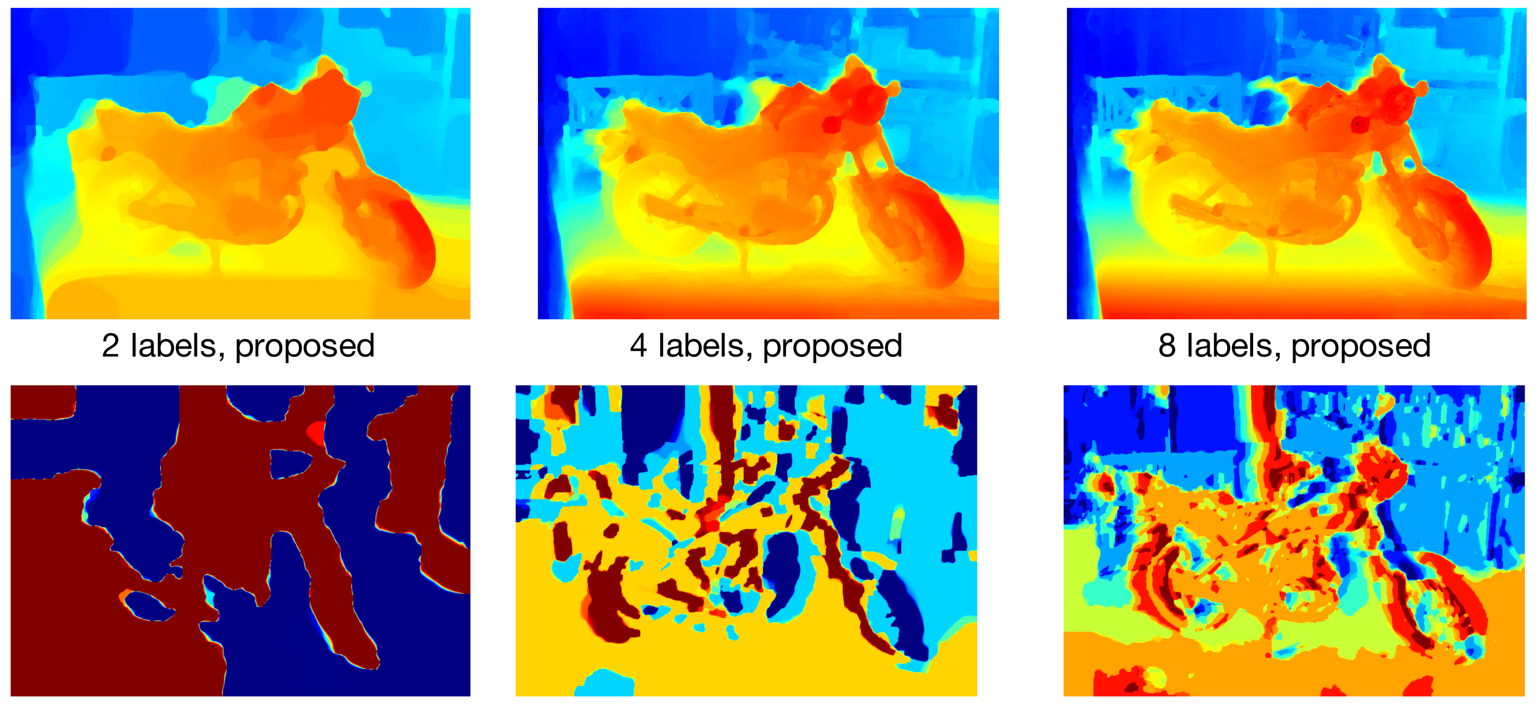
Numerical Experiments

- **Stereo matching, memory and runtime comparison:**



One of the two input images | Baseline [1], 48 labels, 1.49 GB, 52 s | Proposed, 8 labels, 0.49 GB, 30 s

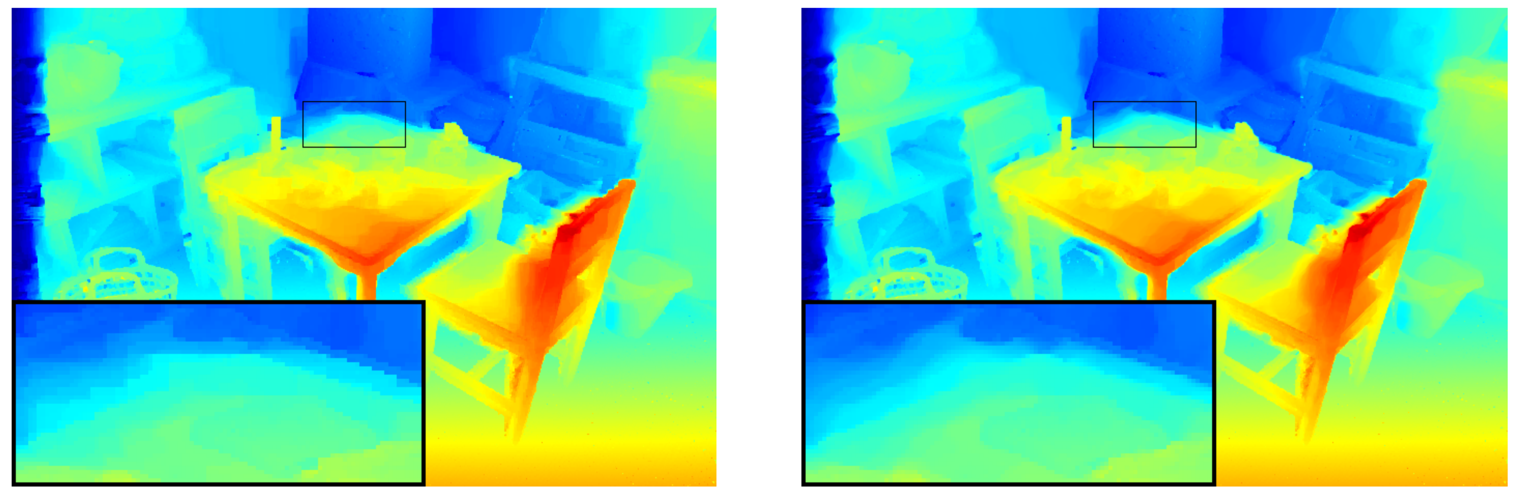
- **Stereo matching, 2-8 labels are often enough:**



2 labels, proposed | 4 labels, proposed | 8 labels, proposed

2 labels, baseline [1] | 4 labels, baseline [1] | 8 labels, baseline [1]

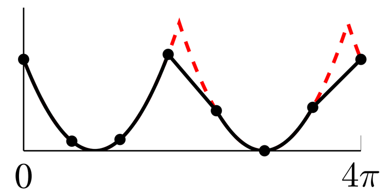
- **Isotropic vs anisotropic regularization:**

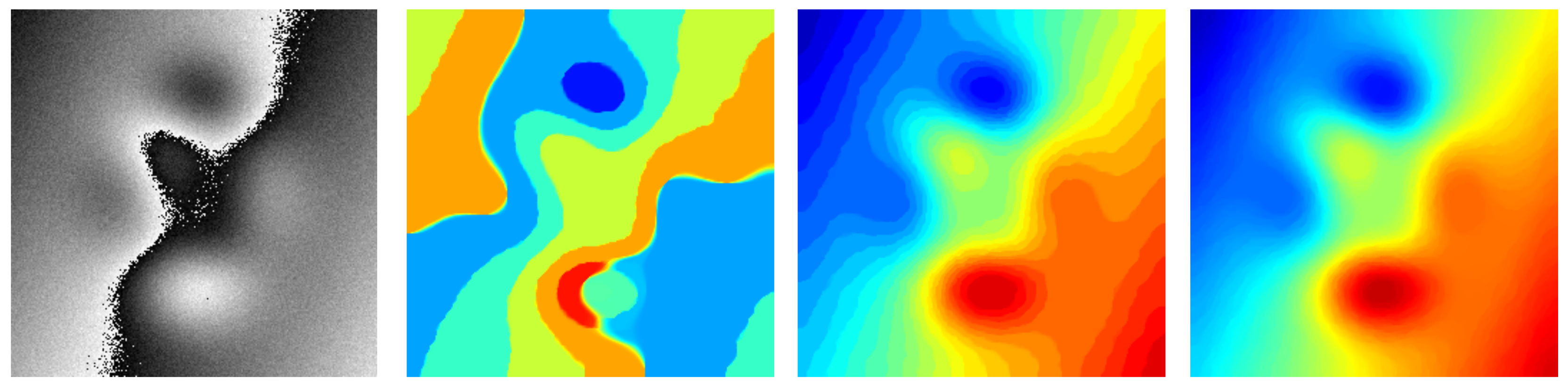


Proposed, anisotropic regularizer (expected output of [3]) | Proposed, isotropic regularization

- **Phase unwrapping:**

$$\rho(x, u(x)) = d_{\mathbb{S}^1}(u(x), f(x))^2$$





Input | Baseline [1], 8 labels | Baseline [1], 32 labels | Proposed, 8 labels

References

[1] T. Pock, D. Cremers, H. Bischof, A. Chambolle. Global solutions of variational models with convex regularization. SIAM J. Imaging Sci. 2010

[2] T. Pock, D. Cremers, H. Bischof, A. Chambolle. An algorithm for minimizing the piecewise smooth Mumford-Shah functional, ICCV, 2009

[3] C. Zach, P. Kohli. A convex discrete-continuous approach for Markov random fields. ECCV, 2012

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